SCEGGS Darlinghurst

Name: ____________

**Term 1, 2003**
**Monday 17th March**

# EXTENSION 1 MATHEMATICS

**Task Weighting : 10 %**

## General Instructions

- Time allowed - 70 minutes
- Write your name at the top of each page
- Start each question on a new page
- Attempt **all** questions.
- Marks may be deducted for careless or badly arranged work
- Approved calculators should be used
- Mathematical templates and geometrical equipment may be used.

| | |
|---|---|
| **Question 1** Reas 1 /1 | /12 |
| **Question 2** Reas 4 /4 Com 1 /1 | /12 |
| **Question 3** | /12 |
| **Question 4** Reas 5 /5 | /12 |
| **TOTAL** | **/48** |



QUESTION 1 (12 marks) Marks

(a) Expand and simplify **2**

$$(p+q)^2-(p-q)^2$$

(b) Factorise fully

(i) $4x^3-12x^2-x+3$ **3**

(ii) $6p^2-5pq-4q^2$ **1**

$-24$ $-8$ $+3$
$6p-8q$ $6p+3q$
$(3p-4q$ $2p+q$

(c) Simplify fully **4**

$$\frac{x^2-9}{x^4-27x} \div \frac{x+3}{x^2+3x+9}$$

(d) (i) Expand $\left(x+\frac{1}{x}\right)^2$ **1**

(ii) Given that $x+\frac{1}{x}=3$ **1R**

use part (i) to evaluate $x^2+\frac{1}{x^2}$

without attempting to find the value of x.

QUESTION 2 (12 marks) START A NEW PAGE Marks

(a) Solve for x 2

$$\frac{1}{2x} - \frac{2}{3} = 1 - \frac{1}{3x}$$

(b) Solve for x 3

$$\frac{x+1}{x-1} \le 2$$

(c) Solve simultaneously for $a, b, c$. 4R

$$3a - 2b - c = -8$$
$$5a + b + 3c = 23$$
$$4a + b - 5c = -18$$

(d) Katie was asked to find the values of $x$ for which $x \times x \le x + x$.
To do this, she decided to solve the inequality $x^2 \le 2x$.
She divided by $x$ and concluded that the solution was $x \le 2$.

Explain why Katie was incorrect in solving the inequality this way. 1C

Show the correct solution. 2



QUESTION 3 (12 marks) START A NEW PAGE Marks

(a) Given that $\tan\theta = -\frac{5}{12}$ and that $\theta$ is obtuse,

find $\sin\theta$ and $\sec\theta$. 3

(b) Solve for $-180° \le \theta \le 180°$,

$$\tan\theta = -1$$ 2

(c) Solve for $0° \le \theta \le 360°$,

$$\sec^2\theta = 2$$ 3

(d) Solve for $0° \le \theta \le 360°$,

$$\sin 2\theta - 2\cos^2\theta = 0$$ 4

QUESTION 4 (12 Marks) START A NEW PAGE Marks

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \cos^2\theta - (1 - \cos^2$
$= \cos^2\theta - 1 + \cos^2$
$= 2\cos^2\theta - 1$

(a) Prove the identity

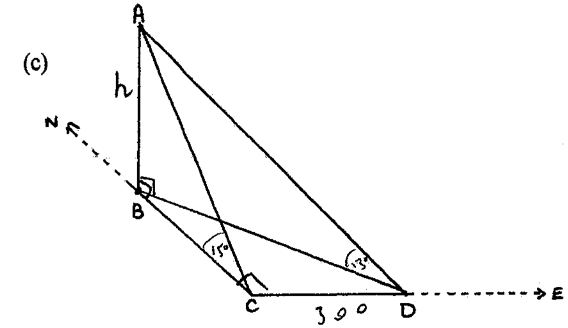
$$\frac{2\cos^3\theta - \cos\theta}{\sin\theta\,\cos^2\theta - \sin^3\theta} = \cot\theta$$ 3(2R)

(b) By expressing $15°$ as either $(45-30)°$ or $(60-45)°$, prove that

$$\sin 15° = \frac{\sqrt{6}-\sqrt{2}}{4}$$ 3



(c)




B, C, D are points on level ground, with D a distance of 300 metres due east of C and B due north of C.
A vertical mast AB stands at B.
At C, the angle of elevation to the top of the mast is 15° and at D, the angle of elevation of A is 13°.

(i) In $\Delta ABC$, show that $BC = h\cot 15°$ 1

(ii) Similarly, show that $BD = h\cot 13°$ 1

(iii) Show that

$$h = \frac{300}{\sqrt{\cot^2 13° - \cot^2 15°}}$$ 3R

(iv) Hence find the height of the mast to the nearest metre. 1

END OF EXAMINATION

Extension 1 Year 11

Solutions.

1)a) $(p+q)^2-(p-q)^2$

$=p^2+2pq+q^2-(p^2-2pq+q^2)$ ✓

$=\cancel{p^2}+2pq+\cancel{q^2}-\cancel{p^2}+2pq-\cancel{q^2}$

$=4pq$ ✓

b) i) $4x^3-12x^2-x+3$

$=4x^2(x-3)-1(x-3)$ ✓

$=(x-3)(4x^2-1)$ ✓

$=(x-3)(2x-1)(2x+1)$ ✓

ii) $6p^2-5pq-4q^2$

$3p \quad -4q$

$2p \quad +q$

$=(3p-4q)(2p+q)$ ✓

c)

$\dfrac{x^2-9}{x^4-27x} \div \dfrac{x+3}{x^2+3x+9}$

✓ ✓

$=\dfrac{(x-3)\cancel{(x+3)}}{x(x^3-27)} \times \dfrac{x^2+3x+9}{\cancel{x+3}}$

$=\dfrac{(x-3)(x^2+3x+9)}{x(x-3)(x^2+3x+9)}$ ✓

$=\dfrac{1}{x}$ ✓

d) i) $\left(x+\frac{1}{x}\right)^2$

$=x^2+2x\cdot\frac{1}{x}+\frac{1}{x^2}$

$=x^2+2+\frac{1}{x^2}$ ✓

ii) $x^2+\frac{1}{x^2}+2=\left(x+\frac{1}{x}\right)^2$

$x^2+\frac{1}{x^2}+2=3^2$

$x^2+\frac{1}{x^2}=9-2$

$=7$ ✓ (1R)

2) a) $\dfrac{1}{2x}-\dfrac{2}{3}=1-\dfrac{1}{3x}$

$\dfrac{3-4x}{6x}=\dfrac{6x-2}{6x}$

$3-4x=6x-2$ ✓

$-10x=-5$

$x=\dfrac{-5}{-10}$

$x=\dfrac{1}{2}$ ✓

b) $\dfrac{x+1}{x-1} \le 2$

undefined for $x=1$ ✓

$\dfrac{x+1}{\cancel{x-1}}\times(x-1)^{\cancel{2}} \le 2(x-1)^2$

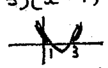
$(x+1)(x-1) \le 2(x^2-2x+1)$

$x^2-1 \le 2x^2-4x+2$

$0 \le x^2-4x+3$

$x^2-4x+3 \ge 0$ ✓

$(x-3)(x-1) \ge 0$




$x \le 1,\ x \ge 3$ ✓

Solution

$x<1,\ x \ge 3$

c)

$3a-2b-c=-8$ ①

$5a+b+3c=23$ ②

$4a+b-5c=-18$ ③

②×2 $\quad 10a+2b+6c=46$ ④

③×2 $\quad 8a+2b-10c=-36$ ⑤

$\quad 3a-2b-c=-8$ ①

④−⑤ $\quad 2a+16c=82$ ⑥

⑤+① $\quad 11a-11c=-44$ ⑦ ✓

⑥×11 $\quad 22a+176c=902$ ⑧

⑦×2 $\quad 22a-22c=-88$ ⑨

⑧−⑨ $\quad 198c=990$

$c=5$ ✓

Substitute into ⑧

$22a+880=902$

$22a=22$

$a=1$ ✓

Substitute into ②

$5+b+15=23$

$b+20=23$

$b=3$ ✓

Solution

$a=1$

$b=3$

$c=5$

(4R)

d) It is incorrect to divide both sides by $x$.

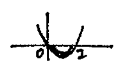
Part of the solution is lost this way.

She needs to factorize the quadratic and then solve the inequality. ✓ (1C)

$x^2 \le 2x$

$x^2-2x \le 0$

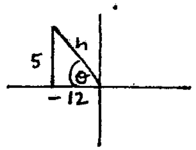
$x(x-2) \le 0$ ✓




$0 \le x \le 2$ ✓

a) $\tan\theta = -5/12$, $\theta$ is obtuse

$\theta$ lies in quadrant 2




By Pythagoras

$h^2 = 5^2 + 12^2$

$= 25 + 144$

$= 169$

$h = 13$

$\sin\theta = \frac{5}{13}$

$\sec\theta = \frac{1}{\cos\theta}$

$= \frac{1}{-\frac{12}{13}}$

$= -\frac{13}{12}$

b) $\tan\theta = -1$

$\theta$ lies in Quadrants 2 and 4

Acute angle

$\tan\theta = 1$

$\theta = 45°$

$\therefore \theta = (180-45)°, (360-45)°$

$= 135°, 315°$

for $-180° \le \theta \le 180°$

$= 135°, -45°$

c) $\sec^2\theta = 2$

$\sec\theta = \pm\sqrt{2}$

$\sec\theta = \sqrt{2}$

$\frac{1}{\cos\theta} = \sqrt{2}$

$\cos\theta = \frac{1}{\sqrt{2}}$

Quad 1 & 4

$\theta = 45°, (360-45)°$

$= 45°, 315°$

$\sec\theta = -\sqrt{2}$

$\frac{1}{\cos\theta} = -\sqrt{2}$

$\cos\theta = -\frac{1}{\sqrt{2}}$

Quad 2 and 3

$\theta = (180-45)°, (180+45)°$

$= 135°, 225°$

d) $\sin 2\theta - 2\cos^2\theta = 0$

$2\sin\theta\cos\theta - 2\cos^2\theta = 0$

$2\cos\theta(\sin\theta - \cos\theta) = 0$

$2\cos\theta = 0$

$\cos\theta = 0$

$\theta = 90°, 270°$

$\sin\theta - \cos\theta = 0$

$\sin\theta = \cos\theta$

$\frac{\sin\theta}{\cos\theta} = 1$

$\tan\theta = 1$

quad 1 and 3

$\theta = 45°, (180+45)°$

$= 45°, 225°$

a) LHS $= \frac{2\cos^3\theta - \cos\theta}{\sin\theta\cos^2\theta - \sin^3\theta}$

$= \frac{\cos\theta(2\cos^2\theta - 1)}{\sin\theta(\cos^2\theta - \sin^2\theta)}$

$= \frac{\cos\theta(2\cos^2\theta - 1)}{\sin\theta(\cos^2\theta - (1-\cos^2\theta))}$

$= \frac{\cos\theta(2\cos^2\theta - 1)}{\sin\theta(2\cos^2\theta - 1)}$ (2R)

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$

$=$ RHS

OR

$\frac{\cos\theta(2\cos^2\theta - 1)}{\sin\theta(\cos^2\theta - \sin^2\theta)}$

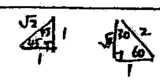
$= \frac{\cos\theta \cdot \cos 2\theta}{\sin\theta \cdot \cos 2\theta}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$

$=$ RHS

b) $\sin 15°$




$= \sin(45° - 30°)$

$= \sin 45°\cos 30° - \cos 45°\sin 30°$

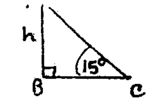
$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$

$= \frac{\sqrt{3} - 1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$

i) In $\Delta ABC$

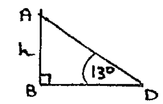



$\tan 15° = \frac{h}{BC}$

$BC = \frac{h}{\tan 15°}$

$BC = h\cot 15°$

ii) In $\Delta ABD$

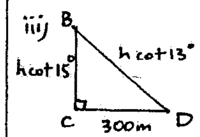



$\tan 13° = \frac{h}{BD}$

$BD = \frac{h}{\tan 13°}$

$BD = h\cot 13°$

iii)




By Pythagoras

$h^2\cot^2 13° = 300^2 + h^2\cot^2 15°$

$h^2(\cot^2 13° - \cot^2 15°) = 300^2$

$h^2 = \frac{300^2}{(\cot^2 13° - \cot^2 15°)}$

$h = \sqrt{\frac{300^2}{\cot^2 13° - \cot^2 15°}}$

$= \frac{300}{\sqrt{\cot^2 13° - \cot^2 15°}}$ (3R)

iv) $h \doteqdot 136.455\ldots$

$h \doteqdot 136$ m (to nearest metre)