
Centre Number

Student Number

SCEGGS Darlinghurst

2009

Preliminary Course
Assessment Task 3

Mathematics Extension 1

Outcomes Assessed: PE3, PE4, PE6
Task Weighting: 20%

General Instructions

- Time allowed – 1 hour
- This paper has **four** questions
- Write your Student Number at the top of each page
- Attempt **all** questions
- Write using blue or black pen
- Answer all questions on the pad paper provided
- Draw all diagrams using a pencil and ruler
- Begin each question on a new page
- Marks will be deducted for careless or badly arranged work
- Approved scientific calculators and mathematical templates may be used

Total marks –

- Attempt Questions 1 – 4

Question	Reasoning	Calc Communication	Marks
1			10
2			10
3			10
4			9
TOTAL			/39

TABLE OF STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}; \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

Note: $\ln x = \log_e x, \quad x > 0$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}} dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

Question 1 (10 marks)

Marks

a) Write down the expansion of $\tan(\alpha + \beta)$ and hence show that $\tan 75^\circ = 2 + \sqrt{3}$ — 3

b) By using the substitution $t = \tan\frac{\theta}{2}$, or otherwise, show that — 2

$$\frac{1 - \cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$$

c)

i) Find the Cartesian equation of the parabola $x = 12t$, $y = 6t^2$ — 1

ii) Find the focus and directrix of the parabola — 2

iii) The point $(3, \frac{3}{8})$ lies on the parabola. Find the value of the parameter 't' at this point — 1

iv) Sketch this parabola showing all important information — 1

Question 2 (10 marks)

Start a new page

Marks

a)

i) Write $\sin x + \sqrt{3}\cos x$ in the form $r\sin(x + \alpha)$ — 2

ii) Hence solve the equation $\sin x + \sqrt{3}\cos x = 1$ for $0^\circ \le x \le 360^\circ$ — 2

b) A curve has parametric equations $x = 3\sin t$, $y = 3\cos t$. Find the Cartesian equation of this curve. — 2

c) Tangents are drawn to the parabola $x^2 = 8y$ from an external point P(4, 0). Find:

i) The equation of the chord of contact. — 1

ii) The coordinates of the points of contact. — 1

iii) The equations of the two tangents from P. — 2

Question 3 (10 marks)
Start a new page | Marks

a) Show that $\dfrac{2\sin^3 A + 2\cos^3 A}{\sin A + \cos A} = 2 - \sin 2A$ (if $\sin A + \cos A \neq 0$) 3

b) T $(2at, at^2)$ is a point on the parabola $x^2 = 4ay$ with focus F.

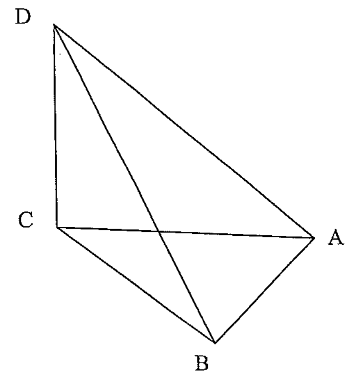
i) Show that the tangent to the parabola at T has gradient t and equation $tx - y - at^2 = 0$ 2

ii) The tangent to the parabola at T cuts the x-axis at X and the y-axis at Y. Show that FX is the perpendicular bisector of TY. 3

c) Find the general solution to the equation $\sqrt{3}\tan x + 1 = 0$ 2

Question 4 (9 marks)
Start a new page | Marks

a) A, B and C are three points on horizontal ground such that A is due East of C and B is 12m due South of A. There is a vertical flagpole of height h metres at point C. From A and B the angles of elevation to the top D of the flagpole are 50° and 20° respectively.



i) Find expressions for AC and BC in terms of h. 2

ii) Hence find the height of the flagpole correct to the nearest cm 2

b) P$(2ap, ap^2)$ is a variable point on the parabola $x^2 = 4ay$. The equation of the tangent at P is $y = px - ap^2$. (Do Not Prove This)

i) The tangent to the parabola at P meets the y-axis at T, find the coordinates of T. 1

ii) The line through P parallel to the axis of the parabola meets the directrix at R, find the coordinates of R. 1

iii) If M is the midpoint of T and R, find the coordinates of M. 1

iv) Show that the equation of the locus of M is a parabola and give the coordinates of its vertex. 2

End of Paper

Preliminary Extension Task 3 2009 Solutions

Question 1

a) $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$ ✓

$\tan 75° = \tan(30°+45°) = \dfrac{\tan 30° + \tan 45°}{1-\tan 30° \times \tan 45}$

$= \left(\frac{1}{\sqrt3} + 1\right) \div \left(1 - \frac{1}{\sqrt3}\times 1\right)$ ✓

$= \dfrac{\sqrt3+1}{\cancel{\sqrt3}} \times \dfrac{\cancel{\sqrt3}}{\sqrt3-1}$

$= \dfrac{\sqrt3+1}{\sqrt3-1} \times \dfrac{\sqrt3+1}{\sqrt3-1}$

$= \dfrac{3+2\sqrt3+1}{2}$ ✓

$= 2+\sqrt3$

This mark Reas 1

b) $t = \tan\frac{\theta}{2}$ $\quad \sin\theta = \dfrac{2t}{1+t^2}$ $\quad \cos\theta = \dfrac{1-t^2}{1+t^2}$

$\dfrac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$ ✓

$LHS = \left(1 - \dfrac{1-t^2}{1+t^2}\right) \div \dfrac{2t}{1+t^2}$

$= \dfrac{1+t^2-1+t^2}{\cancel{1+t^2}} \times \dfrac{\cancel{1+t^2}}{2t}$

$= \dfrac{2t^2}{2t}$ ✓

$= t$

$= \tan\frac{\theta}{2}$

$= RHS$

Reas 1

c) $x = 12t$, $y = 6t^2$

i) $t = \frac{x}{12} \Rightarrow y = 6\left(\frac{x}{12}\right)^2$ ✓

$y = \dfrac{x^2}{24}$

$24y = x^2$

ii) vertex $(0,0)$ $\quad \therefore$ focus $(0,6)$ ✓

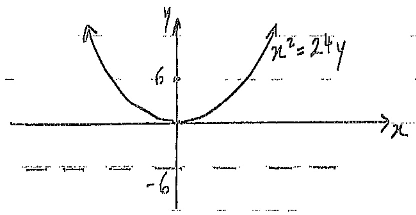
focal length: $4a = 24$ $\quad$ directrix $y=-6$ ✓

$a = 6$

iii) at $x = 3$ $\quad t = \frac{x}{12}$

$t = \frac{1}{4}$ ✓

Reas 1

iv) 

✓

Comm 1

Question 2

a) i) $\sin x + \sqrt{3}\cos x$

$\sin\alpha = \sqrt{3}$

$\cos\alpha = 1$ $\qquad \tan\alpha = \sqrt{3}$ $\qquad r = \sqrt{1^2 + \sqrt{3}^2}$

$\alpha = 60^\circ$ $\qquad r = 2$

$\therefore \sin x + \sqrt{3}\cos x = 2\sin(x + 60^\circ)$

ii) $2\sin(x+60^\circ) = 1$

$\sin(x+60^\circ) = \frac{1}{2}$ $\qquad$ Q1, 2 $\qquad 0^\circ \le x \le 360^\circ$

$60^\circ \le x + 60^\circ \le 420^\circ$

$x + 60^\circ = 30^\circ,\ 180^\circ - 30^\circ,\ 30^\circ + 360^\circ$

$x + 60^\circ = 30,\ 150,\ 390$

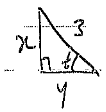
$\therefore x = -30,\ 90^\circ,\ 330^\circ$

$\uparrow$ not a solution

$\therefore x = 90^\circ, 330^\circ$ ✓✓

b) $x = 3\sin t$, $y = 3\cos t$

$\sin t = \frac{x}{3}$ $\qquad \cos t = \frac{y}{3}$



$\therefore x^2 + y^2 = 9$ ✓✓

c) $x^2 = 8y$ $\qquad P(4, 0)$ $\qquad a = 2$

i) $xx_0 = 2a(y + y_0)$

$4x = 4(y)$

$x = y$

ii) simultaneous eqns $\quad x^2 = 8y$ ①

$x = y$ ②

substitute ② into ①

$y^2 = 8y$

$y^2 - 8y = 0$

$y(y-8) = 0 \quad \therefore y = 0,\ y = 8$

at $y = 0$, $x = 0$ $\qquad$ at $y = 8$, $x = 8$

$A(0,0)$ $\qquad B(8,8)$

iii) Eqns of tangents

Tangent at A: $m_A = \frac{0}{4} = 0$ $\qquad$ Tangent at B: $m_B = \frac{8-0}{8-4} = 2$

$\therefore y - 0 = 0(x - 0)$ $\qquad y - 8 = 2(x - 8)$

$y = 0$ ✓ $\qquad y - 8 = 2x - 16$

$0 = 2x - y - 8$ ✓

Question 3

a) $$\frac{2\sin^3 A + 2\cos^3 A}{\sin A + \cos A} = 2 - \sin 2A$$

$$LHS = \frac{2\cancel{(\sin A + \cos A)}(\sin^2 A - \sin A\cos A + \cos^2 A)}{\cancel{\sin A + \cos A}}$$

$$= 2(\sin^2 A + \cos^2 A - \sin A\cos A)$$

$$= 2(1 - \sin A\cos A)$$

$$= 2 - 2\sin A\cos A$$

$$= 2 - \sin 2A$$

$$= RHS$$

Reas 3

b) $T(2at, at^2)$ $\quad x^2 = 4ay$ $\quad F(0, a)$

i) Parametric OR Cartesian $y = \frac{x^2}{4a}$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{2at}{2a} = t$$

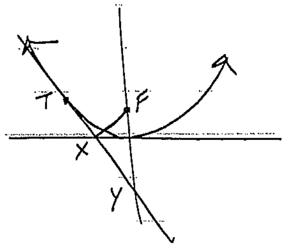
$$\frac{dy}{dx} = \frac{x}{2a} = \frac{2at}{2a} = t \quad \text{at } x = 2at$$

Calc 2

Eqn: $y - at^2 = t(x - 2at)$

$y - at^2 = xt - 2at^2$

$0 = tx - y - at^2$



ii) at X $y = 0$ $\therefore$ $tx - at^2 = 0$

$x = at$ $\qquad X(at, 0)$

at Y $x = 0$ $\qquad y + at^2 = 0$

$y = -at^2$ $\qquad Y(0, -at^2)$

Gradients $\quad M_{FX} = \frac{a - 0}{0 - at} = -\frac{1}{t} \qquad M_{TY} = \frac{at^2 + at^2}{2at - 0} = t$

since $M_{FX} \times M_{TY} = -1$ $\quad FX \perp TY$

Midpoint of TY $\quad x = \frac{2at + 0}{2} \qquad y = \frac{at^2 - at^2}{2}$

$x = at \qquad y = 0$

M.P. $(at, 0)$

Reas 3

since X has the same coordinates as the midpoint FX is the perpendicular bisector of TY

c) $\sqrt{3}\tan x + 1 = 0$

$\tan x = -\frac{1}{\sqrt{3}}$ $\quad \therefore$ Q 2, 4

Q2: $x = 180 - 30,\ 360 + 180 - 30,\ \ldots$

$x = 180 - 30,\ 3 \times 180 - 30,\ \ldots$

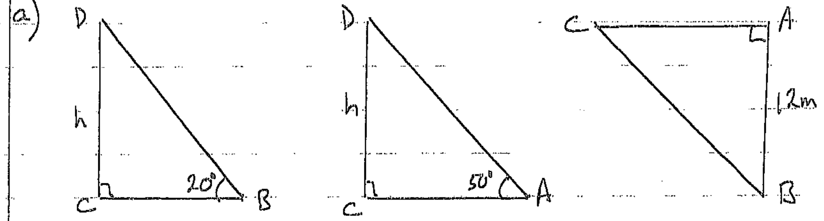
Q4: $x = 360 - 30,\ 360 + 360 - 30^\circ,\ \ldots$

$= 2 \times 180 - 30,\ 4 \times 180 - 30,\ \ldots$

$\therefore$ $x = 180^\circ n - 30^\circ$

Question 4

a)



$\tan 70° = \frac{BC}{h}$ $\qquad$ $\tan 40° = \frac{AC}{h}$

i) $BC = h \tan 70°$ $\qquad$ $AC = h \tan 40°$

ii) By Pythagoras: $12^2 = h^2 \tan^2 70 - h^2 \tan^2 40$

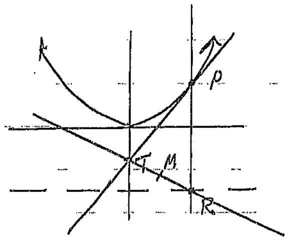
$144 = h^2 (\tan^2 70 - \tan^2 40)$

$h^2 = 21{\cdot}04$

$h = 4{\cdot}59\,m$

b) i) at T $x = 0$

$\therefore y = -ap^2$ $\qquad$ $T(0, -ap^2)$



ii) Directrix: $y = -a$

at R $(2ap, -a)$

iii) M: $x = \frac{2ap}{2}$ $\qquad$ $y = \frac{-a \mp ap^2}{2}$

$M\left(ap, \frac{-a \mp ap^2}{2}\right)$

iv) $x = ap$

$p = \frac{x}{a}$

$y = \frac{-a \mp ap^2}{2}$

$y = \frac{-a \mp \frac{ax^2}{a^2}}{2}$

$y = \frac{-a \mp \frac{x^2}{a}}{2}$

$2y + a = \frac{x^2}{a}$

$2a\left(y + \frac{a}{2}\right) = x^2$

vertex $\left(0, -\frac{a}{2}\right)$