SIMPLE HARMONIC MOTION

EXERCISE 6.8

1. A particle is moving in simple harmonic motion, with displacement at any time t seconds given by $x = 2\cos t$.
   (a) Sketch the graph of its displacement.
   (b) Write down the equation for the velocity of the particle, and sketch the graph of the velocity.
   (c) Find the equation and sketch the graph of the acceleration of the particle.

2. A particle is moving in SHM such that its displacement at any time t seconds is given by $x = 5\sin t$.
   (a) Sketch the graph of its displacement.
   (b) Write down the equation for the velocity of the particle, and sketch the graph of the velocity.
   (c) Find the equation and sketch the graph of the acceleration of the particle.

3. A particle is oscillating about a central point so that its displacement at any time t seconds is given by $x = 4\cos 2t$.
   (a) Sketch the graph of its displacement.
   (b) Find the times when the particle will have maximum displacement, and find this maximum displacement.
   (c) Write down the equation for the velocity of the particle, and sketch the graph of the velocity.
   (d) Find the velocity when the particle is at its maximum displacement.
   (e) Find the equation and sketch the graph of the acceleration of the particle.
   (f) What is the acceleration when the particle is at the origin?

4. A particle moves in SHM so that its acceleration is given by $\ddot{x} = -4x \text{ ms}^{-2}$.
   (a) Show that $x = \cos 2t$ is a formula for the displacement of the particle.
   (b) Find the amplitude and period of the motion.
   (c) Sketch the graph of the particle's displacement.

5. A particle moves in SHM so that its acceleration is given by $\ddot{x} = -9x \text{ ms}^{-2}$.
   (a) Show that $x = 2\cos 3t$ is a formula for the displacement of the particle.
   (b) Find the endpoints of the motion.
   (c) Calculate the velocity when the particle is at the endpoints.
   (d) Find the velocity and acceleration at the centre of the motion.

6. (a) Show that a particle moving in SHM with displacement $x = 7\cos 5t$ has acceleration given by $\ddot{x} = -25x$.
   (b) Find the times at which the particle will have maximum displacement, and find this maximum displacement.
   (c) What is the period of the motion?

7. A particle is moving in SHM with acceleration given by $\ddot{x} = -16x$ ms^{-2}.
   (a) Show that the particle with displacement given by $x = 3\sin 4t$ satisfies this condition for acceleration.
   (b) Find the times when the particle will be at rest.
   (c) Find the displacement and acceleration at these times.

8. A particle is moving in SHM with displacement, in metres, over time t seconds, given by $x = 2 \sin 6t$.
   (a) Find its acceleration in terms of x.
   (b) Find the maximum speed of the particle.
   (c) When will the particle be at the origin, and what will its velocity be at these times?
   (d) Find the equation of the velocity of the particle in terms of x.

9. A particle's displacement is given by $x = 2 \cos \left(t + \frac{\pi}{4}\right)$ m at time t seconds.
   (a) Show that the particle is moving in SHM (i.e., show that its acceleration is proportional to the displacement).
   (b) Find the times at which the particle will be at the origin.
   (c) Write down the period of the motion.
   (d) Find the maximum displacement.

10. (a) Show that a particle is moving according to SHM if its displacement is given by $x = 5 \cos 3t + 2 \sin 3t$, where x is in metres and t is in seconds.
    (b) Find the maximum speed.

11. A particle is moving in SHM and its acceleration is given by $\ddot{x} = -9x \text{ cms}^{-2}$.
    (a) Show that $x = 4 \cos (3t + \pi)$ is a possible equation for the displacement of the particle.
    (b) What will the exact distance of the particle from the origin be when the velocity is 6 cms^{-1}?
    (c) Write down the amplitude and period of the motion.

12. The velocity of a particle moving in SHM in a straight line is given by $v^2 = 4x - x^2 \text{ ms}^{-1}$, where x is displacement in metres.
    (a) Find the two points between which the particle is oscillating.
    (b) Find the centre of the motion.
    (c) Find the maximum speed of the particle.
    (d) Find the acceration of the particle in terms of x.

13. A particle is moving in SHM with acceleration $\frac{d^2x}{dt^2} = -4x \text{ ms}^{-2}$. If the particle starts at the origin with a velocity of 3 ms^{-1}, find
    (a) the endpoints of its motion
    (b) the exact speed when the particle is 1 m from the origin.

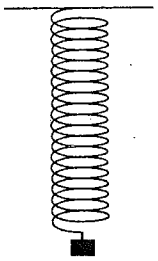
14. A weight is suspended from a spring and pulled down to its maximum displacement of 9 cm, and then let go. Its acceleration is given by $\ddot{x} = -\frac{1}{9}x \text{ cms}^{-2}$. Find
    (a) the equation of its velocity in terms of x
    (b) its exact position when its velocity is 2 cms^{-1}

15. The period of a particle moving in SHM is 6 s and its amplitude is 8 cm. Calculate its velocity and acceleration (correct to 1 decimal place) when the displacement is 5 cm from the centre of the motion.

16. A particle moves in a straight line so that its acceleration at any time is given by $\frac{d^2x}{dt^2} = -9x$. Find its period, amplitude and displacement at time t if initially the particle is 2 cm from the origin and has velocity $2\sqrt{3}$ cms^{-1}.

17. A particle moves in a line so that its acceleration is given by $\frac{d^2x}{dt^2} = 8 - 2x$. Initially, the particle is at the origin and has velocity $3\sqrt{2}$ ms^{-1}.
    (a) Find the interval in which the particle will travel.
    (b) Is the motion of the particle SHM?

18. A weight is oscillating at the end of a spring, with velocity given by $v^2 = 900 - 1600x^2$ cms^{-1}.
    (a) Find the acceleration of the weight with respect to x.
    (b) Find the period of the motion.



    (c) Find the maximum velocity of the weight.

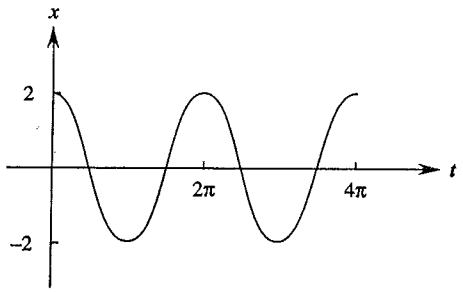
19. (a) If $x = a \sin nt + b \cos nt$, find the acceleration of the particle in terms of t, and show that $\ddot{x} = -n^2x$.
    (b) Find the amplitude and period of the motion.
    (c) Find the maximum velocity.

20. A particle moves in SHM with amplitude 5 cm and period 6 seconds. Find
    (a) the velocity when the particle is 2.5 cm from the centre of motion
    (b) the maximum acceleration

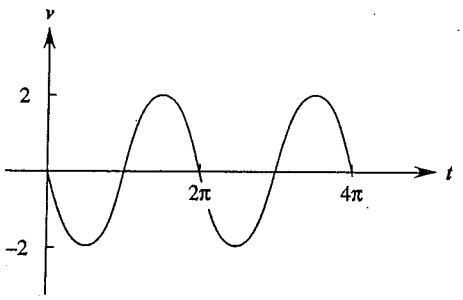
ANSWERS

EXERCISE 6.8

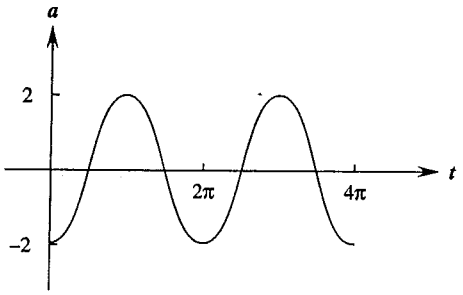
1. (a) $x = 2 \cos t$



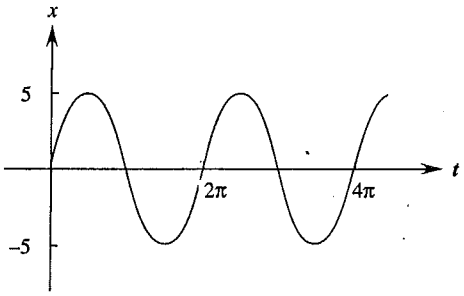
(b) $v = -2 \sin t$



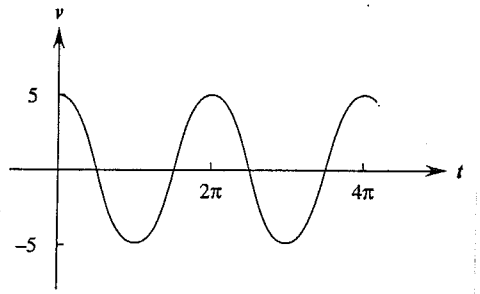
(c) $a = -2 \cos t$



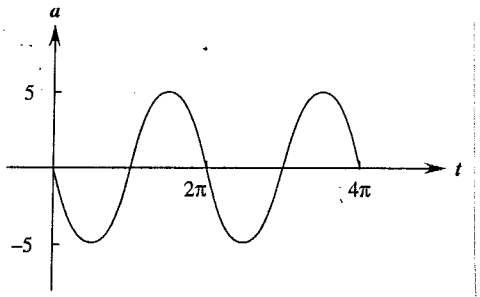
2. (a) $x = 5 \sin t$



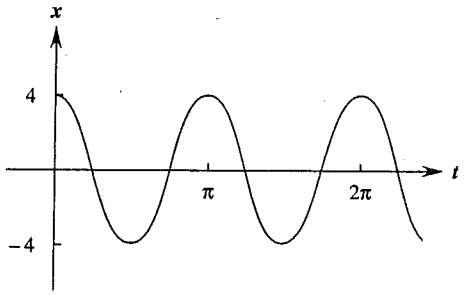
(b) $v = 5\cos t$



(c) $a = \quad = -5\sin t$

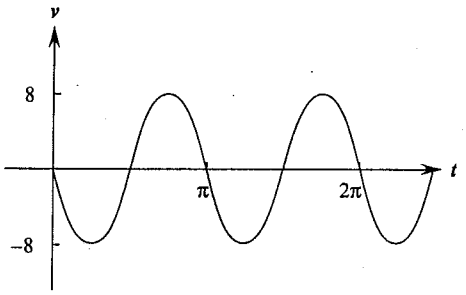


3. (a) $x = 4\cos 2t$



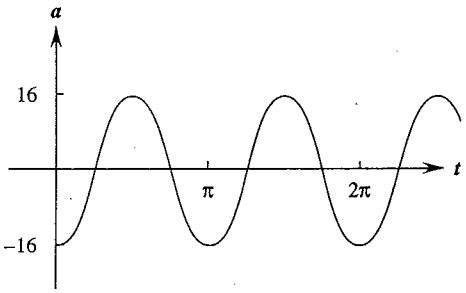
(b) $t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots; x = \pm 4$

(c) $v = -8\sin 2t$



(d) $v = 0$

(e) $a = -16\cos 2t$



(f) $a = 0$

4. (a) $x = \cos 2t$

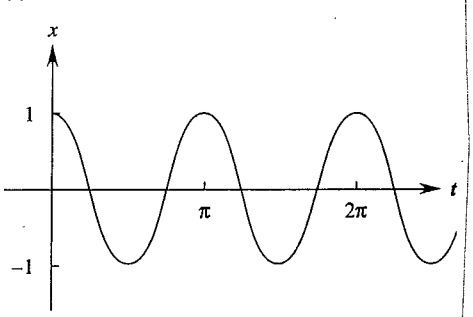
$\dot{x} = -2\sin 2t$

$\ddot{x} = -4\cos 2t$

$= -4x$

(b) amplitude = 1, period = π

(c) $x = \cos 2t$



5. (a) $x = 2\cos 3t$

$\dot{x} = -6\sin 3t$

$\ddot{x} = -18\cos 3t$

$= -9(2\cos 3t)$

$= -9x$

(b) ± 2 (c) $v = 0$ (d) $v = \pm 6$; $a = 0$

6. (a) $x = 7\cos 5t$

$\dot{x} = -35\sin 5t$

$\ddot{x} = -175\cos 5t$

$= -25(7\cos 5t)$

$= -25x$

(b) $t = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \ldots; x = \pm 7$

(c) Period = $\frac{2\pi}{5}$

7. (a) $x = 3\sin 4t$
$\dot{x} = 12\cos 4t$
$\ddot{x} = -48\sin 4t$
$= -16(3\sin 4t)$
$= -16x$

(b) $t = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \ldots$

(c) $x = \pm 3$; $\ddot{x} = \pm 48$ 8. (a) $\ddot{x} = -36x$

(b) $12\ \text{ms}^{-1}$

(c) $t = 0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \ldots$; $\dot{x} = \pm 12\ \text{ms}^{-1}$

(d) $v = \pm\sqrt{144 - 36x^2}$

9. (a) $x = 2\cos\left(t + \frac{\pi}{4}\right)$;

$\dot{x} = -2\sin\left(t + \frac{\pi}{4}\right)$; $\ddot{x} = -2\cos\left(t + \frac{\pi}{4}\right)$

$= -x$, $\therefore$ SHM (b) $t = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \ldots$

(c) 2π (d) $x = \pm 2$

10. (a) $x = 5\cos 3t + 2\sin 3t$
$\dot{x} = -15\sin 3t + 6\cos 3t$
$\ddot{x} = -45\cos 3t - 18\sin 3t$
$= -9(5\cos 3t + 2\sin 3t)$
$= -9x$

(b) $16.2\ \text{ms}^{-1}$

11. (a) $x = 4\cos(3t + \pi)$
$\dot{x} = -12\sin(3t + \pi)$
$\ddot{x} = -36\cos(3t + \pi)$
$= -9[4\cos(3t + \pi)]$
$= -9x$

(b) $2\sqrt{3}$ cm (c) Amplitude $= 4$, period $= \frac{2\pi}{3}$

12. (a) 0 m, 4 m (b) 2 m (c) $2\ \text{ms}^{-1}$
(d) $\ddot{x} = 2 - x$ 13. (a) ± 1.5 m (b) $\sqrt{5}\ \text{ms}^{-1}$

14. (a) $v = \frac{\pm\sqrt{81 - x^2}}{3}$ (b) $x = \pm 3\sqrt{5}$ cm

15. $v = -6.5\ \text{cms}^{-1}$; $a = -5.5\ \text{cms}^{-2}$

16. Period $\frac{2\pi}{3}$, amplitude $\frac{4\sqrt{3}}{3}$;

$x = \frac{4\sqrt{3}}{3}\cos\left(3t - \frac{\pi}{6}\right)$ or

$x = \frac{4\sqrt{3}}{3}\sin\left(3t + \frac{\pi}{3}\right)$

17. (a) Between $x = -1$ and $x = 9$
(b) Yes — centre of motion is $x = 4$
Let $X = x - 4$:

$\frac{d^2x}{dt^2} = 8 - 2x$
$= -2(x - 4)$
$= -2X \quad (n = \sqrt{2})$

18. (a) $a = -1600x$ (b) Period $= \frac{\pi}{20}$
(c) $30\ \text{cms}^{-1}$

19. (a) $\ddot{x} = -an^2\sin nt - bn^2\cos nt$
$= -n^2(a\sin nt + b\cos nt)$
$= -n^2x$

(b) Amplitude: $\sqrt{a^2 + b^2}$; period $= \frac{2\pi}{n}$

(c) $n\sqrt{a^2 + b^2}$ 20. (a) $-\frac{5\sqrt{3}\pi}{6}\ \text{cms}^{-1}$

(b) $\frac{5\pi^2}{9}\ \text{cms}^{-2}$