South Sydney High

Extension Mathematics 1

Function and Circle Geometry
Preliminary Assessment Task

Term 4, 2004	**Date:**	Friday 6th August 2004
Time allowed: 80 minutes	**Assessment:**	Ext. 1 Mathematics

Question 1- (15 marks) **Marks**

(a) If $f(x) = x^4 + bx^2 + cx + d$, find b, c and d, given that $f(x)$ is an even function and $f(0) = 1$, $f(1) = 0$. **2**

(b) A function is defined by the equation $y = 1 + \dfrac{2}{x-4}$.

(i) Draw the graph of $y = 1 + \dfrac{2}{x-4}$ showing the vertical and horizontal asymptotes. **3**

(ii) Hence or otherwise find the values of x for which $1 + \dfrac{2}{x-4} \geq 3$. **2**

(c) Sketch the graphs of $y = 1 + x + |x|$ for $-2 \leq x \leq 2$. **3**

(d)

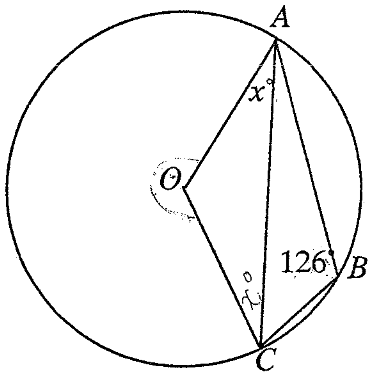
(i) Sketch the graphs of $y = 2|x|$ and $y = |x-3|$, on the same set of axes. **3**

(ii) Shade in the region where $y \leq 2|x|$ and $y \geq |x-3|$ hold simultaneously. **2**

Question 2- (15 marks) **Marks**

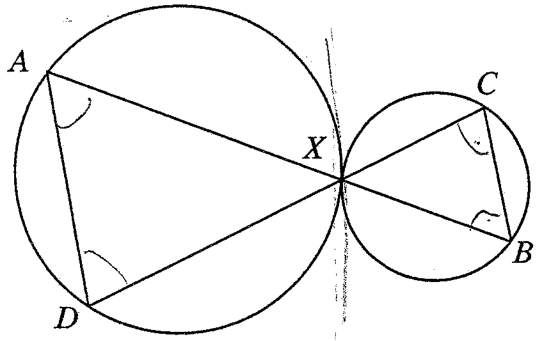
(a) O is the centre of the circle. **5**

$\angle ABC = 126^\circ$, $\angle OAC = x^\circ$



(i) Copy the diagram and find the value of x, giving reasons.

(b) Two circles touch at X.
AXB and CXD are straight lines



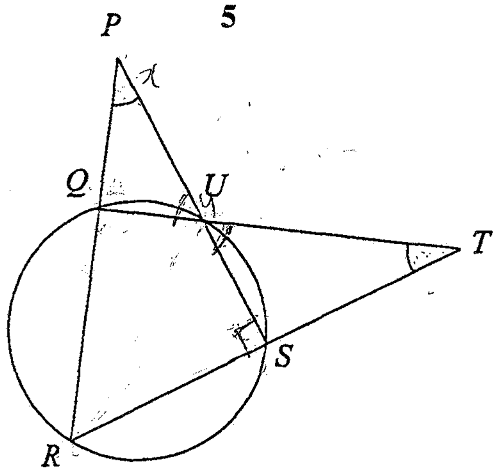
(i) Copy the diagram and draw the common tangent YXZ. **5**

(ii) Prove that $AD // CB$.

(c) In the diagram, $\angle RPS = \angle QTR$. **5**
PQR, PUS, TUQ, TSR are straight lines.

(i) Prove that $\angle UQR = \angle USR$.

(ii) Hence explain why UR is a diameter.



QUESTION 1:

(a) $f(x) = x^4 + bx^2 + cx + d$

$f(x)$ is even

$\therefore f(x) = f(-x)$

ie

$x^4 + bx^2 + cx + d = (-x)^4 + b(-x)^2 + c(-x) + d$

$\cancel{x^4} + \cancel{bx^2} + cx + \cancel{d} = \cancel{x^4} + \cancel{bx^2} - cx + \cancel{d}$

$\therefore 2cx = 0$

$\therefore c = 0$ #

$f(0) = 1$

$\therefore 0^4 + b(0)^2 + c(0) + d = 1$

$\therefore d = 1$ #

$f(1) = 0$

$1^4 + b(1)^2 + c \cdot 1 + d = 0$

$1 + b + c + d = 0$

ie $1 + b + 0 + 1 = 0$ 2

$\therefore b = -2$

$\therefore b = -2, \; c = 0 \quad d = 1$ #

(b) $y = 1 + \frac{2}{x-4} \Rightarrow y = \frac{x-4+2}{x-4}$

$y = \frac{x-2}{x-4}$

$x - 4 \neq 0$

$\therefore x = 4$

$\therefore$ vertical asymptote $x = 4$ #.

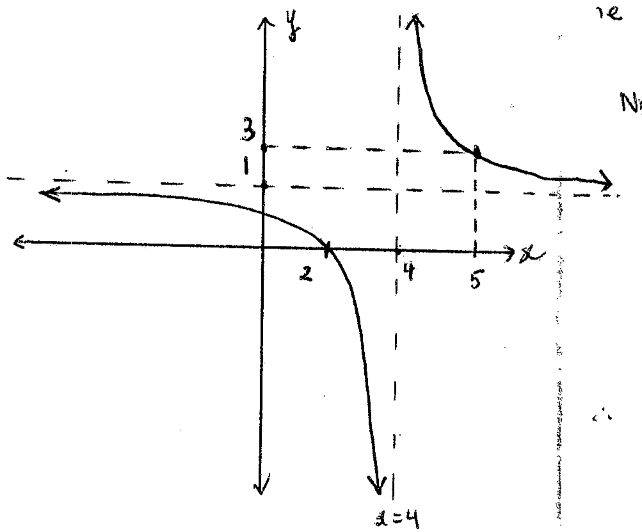
Now to find horizontal asymptotes we find $\lim_{x \to \infty} 1 + \frac{2}{x-4}$

ie as $x \to \infty$

$y = 1 + \frac{2}{\infty}$

$\therefore$ as $x \to \infty$, $y \to 1$ 3

$\therefore$ horizontal asymptote $\Rightarrow y = 1$



* If $y = 1 + \frac{2}{x-4}$

$\Rightarrow y = \frac{x-4+2}{x-4}$

ie $y = \frac{x-2}{x-4}$

* Now, x-intercept occurs at $y = 0$

ie $0 = \frac{x-2}{x-4}$

$\therefore x - 2 = 0$

$\therefore x = 2$ # $\Rightarrow$ x intercept

(ii) $1 + \frac{2}{x-4} \geq 3$

ie $\frac{x-2}{x-4} \geq 3$

Now solving

$\frac{x-2}{x-4} = 3$

$\Rightarrow x - 2 = 3x - 12$ 2

$10 = 2x$

$\therefore x = 5$

$\therefore$ From graph

$\frac{x-2}{x-4} \geq 3$ when $x > 4$ & $x \leq 5$

ie $\{x : 4 < x \leq 5\}$ #

(c) $y = 1 + x + |x| \qquad -2 \leq x \leq 2$

CASE (i) $x > 0$

ie $y = 1 + x + x$

$y = 1 + 2x$

$y = 2x + 1$

$\therefore$ graph $y = 2x + 1 \quad 0 \leq x \leq 2$

Now when $x = 0$, $y = 1$

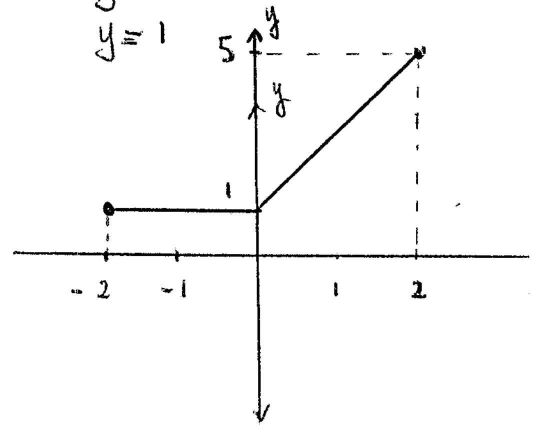
when $x = 2$ $y = 2 \times 2 + 1 = 5$

3

CASE (ii) $x < 0$

ie $y = 1 + x + -x$

$y = 1$



(d) $y \le 2|x|$

ie Sketch $y \le 2x$ for $x > 0$

& $y \le -2x$ for $x < 0$

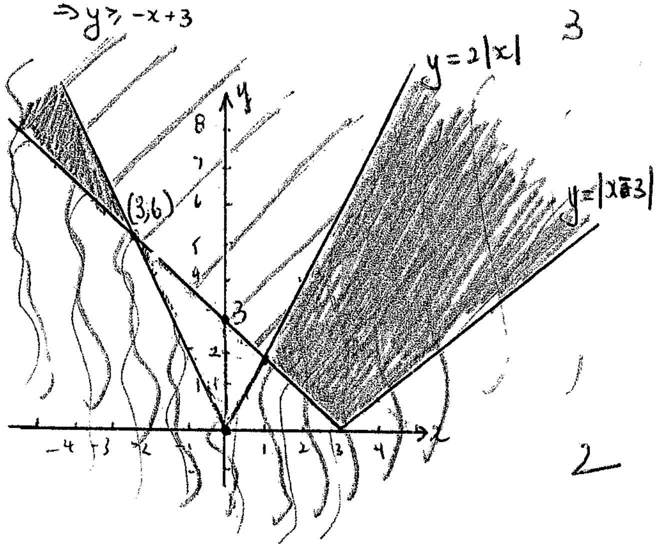
$y \ge |x-3|$

ie $y \ge x-3$ for $x > 3$

& $y \ge -(x-3)$ for $x < 3$

$\Rightarrow y \ge -x+3$

3



2

pts of Intersection

$y = -2x$
$y = -x+3$
$-2x = -x+3$
$-x = 3$
$\therefore x = -3$
ie $(-3, 6)$

$y = 2x$
$y = -x+3$
$2x = -x+3$
$3x = 3$
$\therefore x = 1$
ie $(1, 2)$

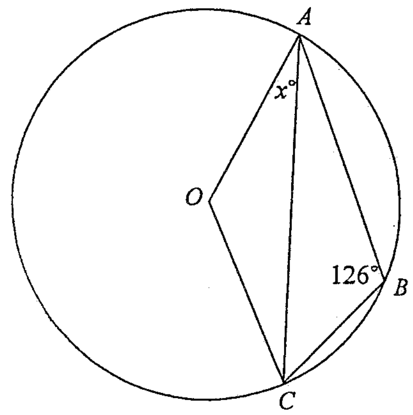
-1 for no pts.

$= (-1)^{k+1}\sin\theta$

$= (-1)^n \sin\theta$ when $n = k+1$ 1 **Total = 3**

SOLUTIONS 3-(

QUESTION 5 2



(a)

Reflex $\angle AOC = 2 \times 126°$ (angle at centre is twice the angle at circumference standing on same arc) 1

$= 252°$ 1

$\angle AOC = 360° - 252°$ (one revolution = 360°)

$= 108°$ 1

ΔAOC is isosceles ($OA = OC$)

$\therefore \angle OAC = \angle OCA$ (angles opposite equal sides) 1

$\therefore x - x - 108 = 180$ (angle sum of triangle AOC is 180°)

$x = 36$ 1 **Total = 5**