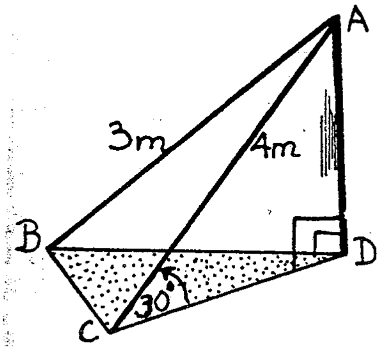
SOUTH SYDNEY HIGH SCHOOL

MATHS – EXT1 WORKSHEETS

HARDER 3D TRIGONOMETRY

1.



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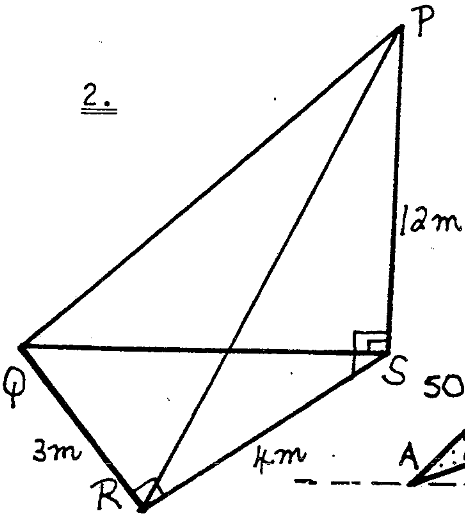

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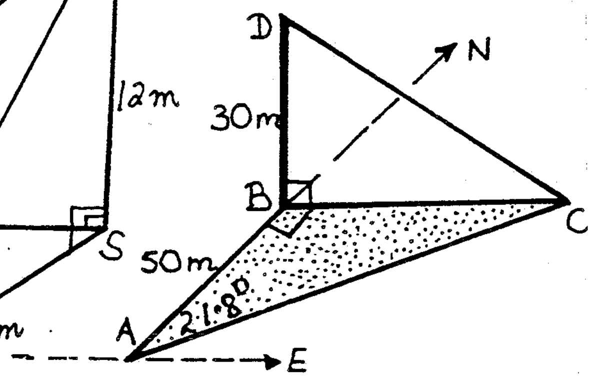

1. AD IS A VERTICAL POST. AB, AC ARE ROPES OF LENGTH 3m, 4m RESPECTIVELY. The rope AC is inclined at 30^0 to the horizontal. Calculate the height of the post and the angle at which the rope AB is inclined to the horizontal (to the nearest degree).

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2. A 3-DIMENSIONAL TIMBER FRAMEWORK IS SKETCHED ABOVE. IT IS KNOWN THAT PS = 12m and is perpendicular to the plane of the triangle QSR which is right-angled at R, and QR = 3m, RS = 4m. Find

(i) the lengths of the joints QS, PQ, PR;

(ii) the size of the angle PQS (to the nearest degree).

Use the results from (i) to show that the angle PRQ is 90^0 and hence calculate the angle QPR, to the nearest degree.

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3. IN THE SKETCH ABOVE, THE POINT C IS N 21.8^0E of A AND B IS 50m DUE NORTH of A.

(a) Calculate the distance of C from B (to nearest m).

(b) A tree of height 30m stands at B. What is the angle of elevation of the top D from C (to nearest degree)?

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4. FROM TWO POINTS C, D ON THE GROUND THE ANGLES OF ELEVATION OF THE TOP A of a tower AB are 34^0 and 29^0 respectively. If B is the foot of the tower and AB = 200m, calculate to 0.1m, the lengths of BC and BD.

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5. A VERTICAL MAST IS 50m HIGH AND IS SUPPORTED BY 4 STAYS EACH FIXED AT A distance of 25m from its foot, their bearings being N, S, E, W of the foot. The tops of the stays are attached to the mast at a point 6m from its top. Find (to the nearest degree) the angle

(i) of elevation of the top of the mast from the foot of a stay;

(ii) each stay makes with the vertical.

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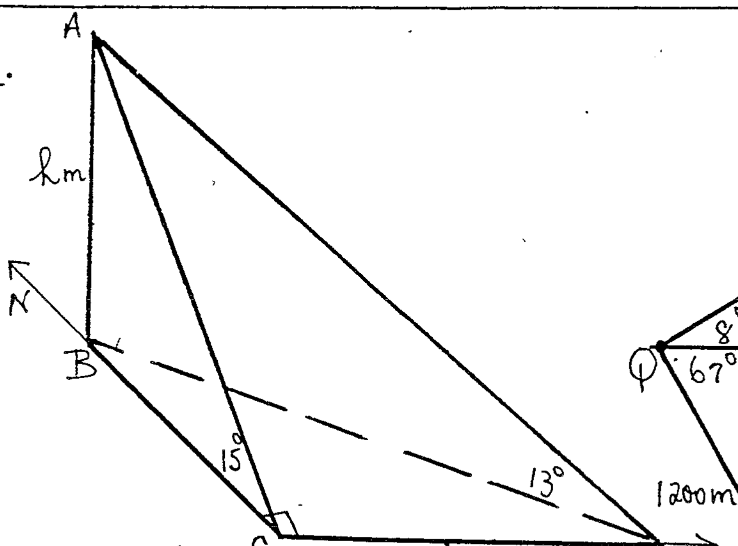
6. A TOWER STANDS AT THE FOOT OF A PLANE WHICH IS INCLINED TO THE HORIZONTAL at an angle of 12^{0}. A line 100m in length is measured straight up the incline from the foot of the tower. It is found that at the end of this line the tower subtends an angle of 62^{0}. Calculate the height of the tower, to the nearest metre.

7.



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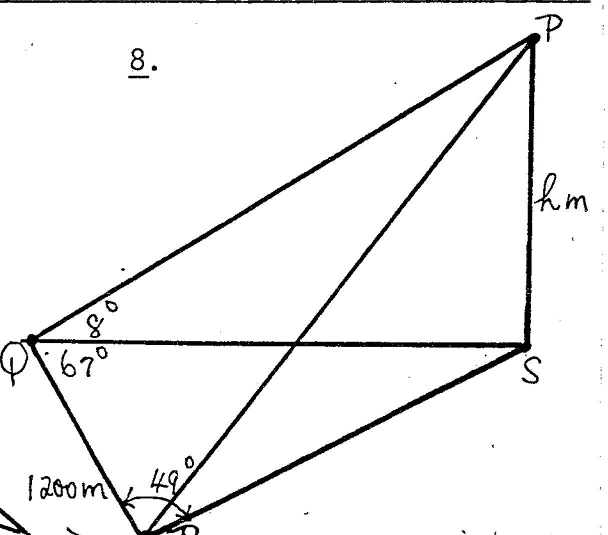

7. IN THE SKETCH, B, C, D ARE POINTS ON LEVEL GROUND, WITH D DISTANT 300m due east of C and B due north of C. A vertical mast AB stands at B. At C the angle of elevation of the top A of the mast is 15^{0} and at D the angle of elevation of A is 13^{0}. If h metres is the height of the mast, show that $h = 300\ (\cot^2 13^{0} - \cot^2 15^{0})^{-\frac{1}{2}}$.

Hence find the height of the mast to the nearest metre.

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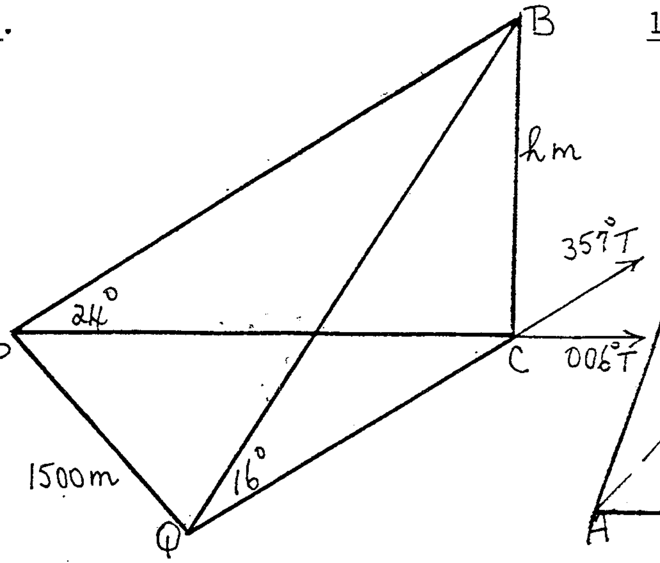
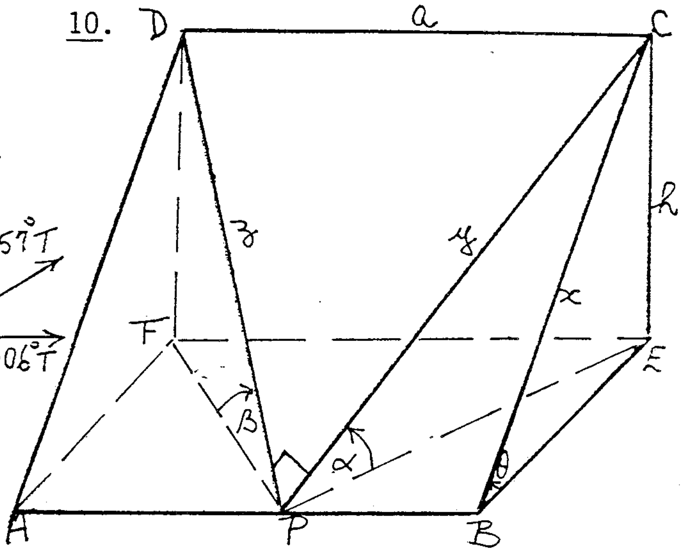
8. IN THE SKETCH ABOVE, Q, R, S ARE ON LEVEL GROUND AND SP IS A TOWER. IT is given that QR = 1200m, $S\hat{Q}R = 67^{0}$, $S\hat{R}Q = 49^{0}$. From Q the elevation of the top P of the tower is 8^{0}. If h m is the height of the tower, show that $h = 1200 \sin 49^{0} \tan 8^{0} / \sin 64^{0}$, and hence find h to the nearest metre.

Also if α is the angle of elevation of P from R, show that $\tan \alpha = \sin 49^{0} \tan 8^{0} / \sin 67^{0}$ and hence find α to the nearest degree.

9.



10.



9. TWO OBSERVERS P, Q 1500 METRES APART TAKE THE BEARING AND ELEVATION OF a balloon B at the same instant. The observer P finds the bearing is 006^{o}T and the angle of elevation 24^{o}, whilst the observer Q finds the bearing to be 357^{o}T and the angle of elevation 16^{o}. Show that if the height BC of the balloon is h metres, then $h = 1500/(\cot^2 24^{o} + \cot^2 16^{o} - 2\cot 24^{o} \cot 16^{o} \cos 9^{o})^{\frac{1}{2}}$ and hence find h to the nearest metre.

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10. THE SKETCH SHOWS TWO STRAIGHT ROADS PC, PD AT RIGHT ANGLES TO ONE ANOTHER, running up a hill ABCD which is inclined at an angle θ to the horizontal. The roads PC, PD are inclined at angles α, β to the horizontal. BC, AD are lines of greatest slope and AB is the horizontal; CE, DF are verticals from C, D to the horizontal plane.

Show that $\sin^2\theta = \sin^2\alpha + \sin^2\beta$.

{*Hint: Use the symbols h, x, y, z, a shown on the figure and prove that* $x = yz/a$.}

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11. A, B ARE TWO POINTS 600m APART ON THE GROUND AND D IS THE TOP OF A TOWER. The angles DAB and DBA are 59^{o} and 54^{o} respectively. The elevation of D from A is 5^{o}. Show that the height h metres of the tower is given by $h = 600 \sin 5^{o} \sin 54^{o} / \sin 67^{o}$ and hence find h to the nearest metre.

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12. AT TWO POINTS A AND B, 400 METRES APART, ON A STRAIGHT HORIZONTAL ROAD, the summit of a hill is observed. At A it is due N with an elevation of 40^{o}; at B it is due W with an elevation of 27^{o}. Find the height of the hill to the nearest 0.1 metre.

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13. FROM ONE POINT ON THE BANK OF A RIVER THE ELEVATION OF A TOWER OF HEIGHT 450m on the same bank is 55^{o}, and from a point on the other bank exactly opposite the first, the elevation is $42^{o}30'$. Show the breadth of the river d metres is given by $d = 450\ (\cot^2 42^{o}30' - \cot^2 55^{o})^{\frac{1}{2}}$ and hence find d to 0.1m.

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14. THE ANGLE OF ELEVATION OF THE SUMMIT OF A MOUNTAIN DUE NORTH IS 14^{o} AND on walking 7000m due west it is found to be 10^{o}. Find the height of the mountain.

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15. TO FIND THE HEIGHT OF AN INACCESSIBLE TOWER, THE ELEVATION OF ITS SUMMIT is observed from a point A due east of the tower and found to be α. The elevation is observed again from a point B due north of the tower and observed to be β. The distance between the points A, B which are on the same level as the foot of the tower, is d metres. Show that, if h metres is the height of the tower, then $d^2 = h^2(\cot^2\alpha + \cot^2\beta)$.

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16. TWO VERTICAL POLES, WHOSE HEIGHTS ARE a AND b, SUBTEND THE SAME ANGLE γ at a point P in the line joining their feet A, B. If Q is another point in the horizontal plane and the angle AQB is a right-angle, then the poles subtend angles of α, β at Q. Prove that $(a + b)^2\cot^2\gamma = a^2\cot^2\alpha + b^2\cot^2\beta$.

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17. THE SIDE OF A HILL FACES DUE SOUTH AND IS INCLINED TO THE HORIZON AT AN angle α. A straight railway upon it is inclined at an angle β to the horizon. If the bearing of the railway is γ^0 east of north, show that $\cos\gamma = \cot\alpha \tan\beta$.

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18. AN AEROPLANE FLIES HORIZONTALLY DUE EAST AT A CONSTANT SPEED OF 240 km/h. From a point P on the ground the bearing of the plane at one instant is 311^0T and 3 minutes later the bearing of the plane is 073^0T whilst its elevation then is 21^0. If h metres is the altitude of the plane, show that $h = 12000 \sin 41^0 \tan 21^0 \operatorname{cosec} 58^0$ and calculate h to the nearest metre.

If α is the angle of elevation of the aeroplane at the first point of observation, show that $\tan\alpha = \sin 41^0 \tan 21^0 \operatorname{cosec} 17^0$ and find α to the nearest minute.

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19. A PERSON WALKING ALONG A STRAIGHT ROAD OBSERVES A SPIRE BEARING 045^0T, the angle of elevation being 5^0. After travelling a distance of 5000m, the spire bears 315^0T and the angle of elevation is 8^0.

Find the height of the spire (to 0.1m), and determine the angle which the road makes with a line bearing 090^0T, (to the nearest minute).

ANSWERS

SET H.D.2 (PAGE 217)

1. 2m; 42^0 2. (i) 5m, 13m, $\sqrt{160} \doteqdot 12.6$m (ii) 67^0; $Q\hat{P}R = 13^0$

3. (a) 20m (b) 56^0 4. 296.5m, 360.8m 5. (i) 63^0 (ii) 30^0

6. 137m 7. 136m 8. 142m; 7^0 9. 1139m

11. 46m 12. 174.2m 13. 376.7m 14. 1745.8m

18. 3564m; $40^0 44^1$ 19. 371.4m; $(45^0 - 31^0 54^1) = 13^0 6^1$

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1. $\sin 30^\circ = \frac{AD}{4}$ $\qquad$ $\sin\theta = \frac{2}{3}$

$\underline{\underline{AD = 2m}}$ $\qquad$ $\underline{\underline{\theta \doteq 42^\circ}}$

2. i) $3^2+4^2 = QS^2$ $\qquad$ $12^2+QS^2 = PQ^2$

$\therefore \underline{\underline{QS = 5m}}$ $\qquad$ $144+5^2 = PQ^2$

$PQ = \underline{\underline{13m}}$

$4^2+12^2 = PR^2$

$PR^2 = 160$

$PR = \underline{\underline{4\sqrt{10}m}}$

ii) $\tan\theta = \frac{12}{5}$

$\theta = 67^\circ$ $\qquad$ $\therefore \underline{\underline{\angle PQS = 67^\circ}}$

3. a) $\angle ACB = 68.2^\circ$ (remaining $\angle$)

$\frac{BC}{\sin 21.8^\circ} = \frac{50}{\sin 68.2^\circ}$

$BC = \underline{\underline{20m}}$

2. $PR^2 = 160$ m

$QR^2 = 9$ m

$PQ^2 = 169$ $\qquad$ $\because PR^2+QR^2 = 169$

$= PQ^2$ $\qquad$ $\therefore \underline{\underline{\angle PRQ \text{ is } 90^\circ}}$

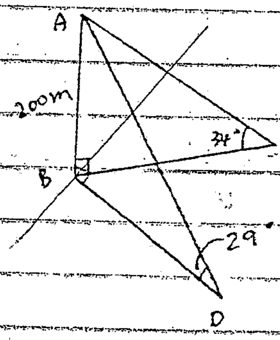
$\sin\theta = \frac{3}{13}$

$\theta = 13^\circ$ $\qquad$ $\therefore \underline{\underline{\angle QPR = 13^\circ}}$

b) $\tan\theta = \frac{30}{20}$

$\theta = 56^\circ$ $\qquad$ $\therefore \underline{\underline{56^\circ}}$

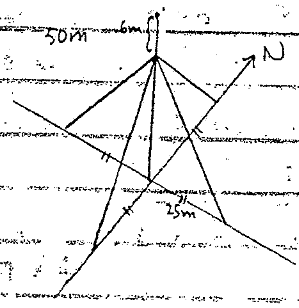
4.



$\tan 29^\circ = \frac{200}{BD}$ $\qquad$ $\tan 34^\circ = \frac{200}{BC}$

$\underline{\underline{BD = 360.8m}}$ $\qquad$ $\underline{\underline{BC = 296.5m}}$

5.



i) $\tan\theta = \frac{50}{25}$

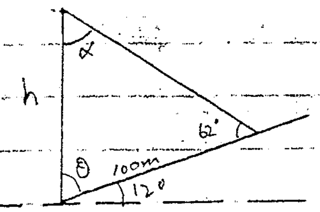
$\underline{\underline{\theta = 63^\circ}}$

$*$ ii) $\tan\theta = \frac{25}{50-6}$

$\tan\theta = \frac{25}{44}$

$\underline{\underline{\cancel{\theta = 30}^\circ}}$

6.



$\theta = 90° - 12° = 78°$

$\therefore \alpha = 40°$ (remaining ∠)

$$\frac{h}{\sin 62°} = \frac{100}{\sin 40°}$$

$$\therefore h = \underline{\underline{137\,m}}$$

※ 7.

$\cot 15° = \frac{BC}{h}$; $\cot 13° = \frac{BD}{h}$

$BC = h \cdot \cot 15°$ $\qquad BD = h \cdot \cot 13°$

$\because BC^2 + CD^2 = BD^2$ (Pyth. Th.)

$h^2 \cot^2 15° + 300^2 = h^2 \cot^2 13°$

$h^2(\cot^2 13° - \cot^2 15°) = 300^2$

$$h = \frac{300}{\sqrt{\cot^2 13° - \cot^2 15°}}$$

$$\therefore \underline{\underline{h = 300 \cdot (\cot^2 13° - \cot^2 15°)^{-\frac{1}{2}}}}$$

Note：佢俾 cot 你，你就用 cot la.... 睇 Question ma...

$$\therefore h = \underline{\underline{136\,m}}$$

8.
- $\angle QSR = 64°$ (remaining ∠)
- $\tan 8° = \frac{h}{QS}$

$QS = \frac{h}{\tan 8°}$

$$\frac{QS}{\sin 49°} = \frac{1200}{\sin 64°}$$

$$\frac{h}{\tan 8°} = \frac{1200 \cdot \sin 49°}{\sin 64°}$$

$$\therefore \underline{\underline{h = \frac{1200 \cdot \sin 49° \cdot \tan 8°}{\sin 64°}}}$$

$\therefore \underline{\underline{h = 142m}}$

$$\because \frac{RS}{\sin 67°} = \frac{1200}{\sin 64°}$$

$$\therefore RS = \frac{1200 \cdot \sin 67°}{\sin 64°}$$

In $\triangle PRS$, $\tan\alpha = \frac{h}{RS}$

$$\tan\alpha = \frac{1200 \cdot \sin 49° \cdot \tan 8°}{\sin 64°} \times \frac{\sin 64°}{1200 \cdot \sin 67°}$$

$$\underline{\underline{\tan\alpha = \frac{\sin 49° \cdot \tan 8°}{\sin 67°}}}$$

9. * $\theta = 6^\circ + (360^\circ - 357^\circ) = 9^\circ$

$\alpha = \theta = 9^\circ$ (vert. opp. ∠s)

$\cot 24^\circ = \frac{PC}{h}$; $\cot 16^\circ = \frac{QC}{h}$

$PC = h \cdot \cot 24^\circ$; $QC = h \cdot \cot 16^\circ$

$\therefore 1500^2 = PC^2 + QC^2 - 2(PC)(QC)\cos 9^\circ$

$1500^2 = h^2 \cot^2 24^\circ + h^2 \cot^2 16^\circ - 2(h \cdot \cot 24^\circ)(h \cdot \cot 16^\circ)\cos 9^\circ$

$1500^2 = h^2(\cot^2 24^\circ + \cot^2 16^\circ - 2 \cdot \cot 24^\circ \cdot \cot 16^\circ \cdot \cos 9^\circ)$

$$h^2 = \frac{1500^2}{\cot^2 24^\circ + \cot^2 16^\circ - 2\cot 24^\circ \cot 16^\circ \cos 9^\circ}$$

$$\therefore h = \frac{1500}{(\cot^2 24^\circ + \cot^2 16^\circ - 2\cot 24^\circ \cot 16^\circ \cos 9^\circ)^{\frac{1}{2}}}$$

$\therefore h = \underline{\underline{1139 m}}$

*10. $AP = \sqrt{z^2 - x^2}$; $PB = \sqrt{y^2 - x^2}$

由 Area 入手 * $\frac{x\sqrt{z^2-x^2}}{2} + \frac{x\sqrt{y^2-x^2}}{2} + \frac{zy}{2} = ax$

$x(\sqrt{z^2-x^2} + \sqrt{y^2-x^2}) + zy = 2ax$

$x(*a - \sqrt{y^2-x^2} + \sqrt{y^2-x^2}) + zy = 2ax$

$ax + zy = 2ax$

$zy = ax$

$x = \frac{zy}{a}$

$\sin\beta = \frac{h}{z}$; $\sin\alpha = \frac{h}{y}$; $\sin\theta = \frac{h}{x}$

$z = \frac{h}{\sin\beta}$; $y = \frac{h}{\sin\alpha}$; $x = \frac{h}{\sin\theta}$

In $\triangle CDP$, $z^2 + y^2 = a^2$

$\frac{h^2}{\sin^2\beta} + \frac{h^2}{\sin^2\alpha} = a^2$

$$a = \sqrt{\frac{h^2}{\sin^2\beta} + \frac{h^2}{\sin^2\alpha}}$$

$$\therefore \frac{h}{\sin\theta} = \frac{\frac{h}{\sin\beta} \cdot \frac{h}{\sin\alpha}}{\sqrt{\frac{h^2}{\sin^2\beta} + \frac{h^2}{\sin^2\alpha}}}$$

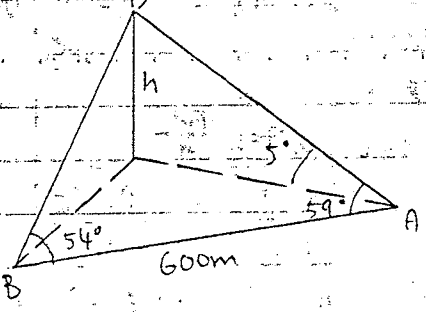
$$\frac{h^2}{\sin^2\theta} = \frac{\frac{h^4}{\sin^2\beta \cdot \sin^2\alpha}}{\frac{\sin^2\alpha\, h^2 + \sin^2\beta\, h^2}{\sin^2\beta \sin^2\alpha}}$$

$$\frac{h^2}{\sin^2\theta} = \frac{h^4}{\sin^2\beta \cdot \sin^2\alpha} \times \frac{\sin^2\beta \sin^2\alpha}{h^2(\sin^2\alpha + \sin^2\beta)}$$

$$\frac{h^2}{\sin^2\theta} = \frac{h^2}{\sin^2\alpha + \sin^2\beta}$$

$\therefore \underline{\underline{\sin^2\theta = \sin^2\alpha + \sin^2\beta}}$

11.



$\sin 5° = \frac{h}{AD}$; $\angle ADB = 67°$ (remain)

$AD = \frac{h}{\sin 5°}$

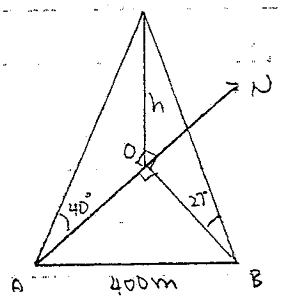
$\therefore \frac{AD}{\sin 54°} = \frac{600}{\sin 67°}$

$\frac{h}{\sin 5° \cdot \sin 54°} = \frac{600}{\sin 67°}$

$\therefore h = \frac{600 \cdot \sin 5° \cdot \sin 54°}{\sin 67°}$

$\therefore h = 46m$

12.



$\tan 40° = \frac{h}{AO}$; $\tan 27° = \frac{h}{BO}$

$AO = \frac{h}{\tan 40°}$ $\quad BO = \frac{h}{\tan 27°}$

$\therefore AO^2 + BO^2 = AB^2$ (Pyth. th.)

$\frac{h^2}{\tan^2 40°} + \frac{h^2}{\tan^2 27°} = 400^2$

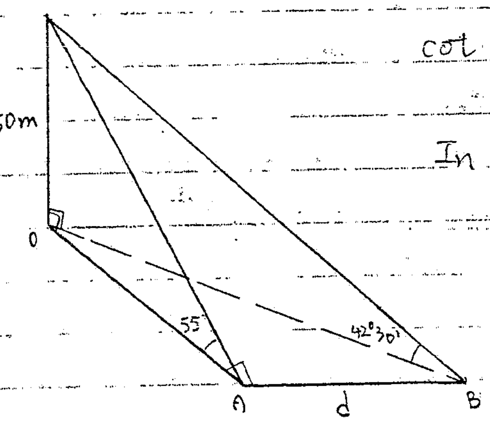
$h^2 \tan^2 27° + h^2 \tan^2 40° = 400^2 \cdot \tan^2 27° \cdot \tan^2 40°$

$h^2 = \frac{400^2 \cdot \tan^2 27° \cdot \tan^2 40°}{\tan^2 27° + \tan^2 40°}$

$\therefore h \doteq 174.2m$

※ 13.

diagram 難draw



$\cot 55° = \frac{AO}{450}$; $\cot 42°30' = \frac{BO}{450}$

$\therefore AO = 450 \cot 55°$ $\quad BO = 450 \cdot \cot 42°30'$

In $\triangle ABO$,

$AO^2 + AB^2 = BO^2$ (Pyth. th.)

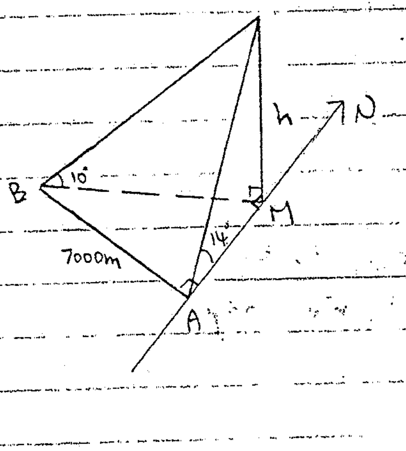
$450^2 \cot^2 55° + d^2 = 450^2 \cot^2 42°30'$

$d^2 = 450^2 (\cot^2 42°30' - \cot^2 55°)$

$\therefore d = 450 (\cot^2 42°30' - \cot^2 55°)^{\frac{1}{2}}$

$\therefore d = 376.7m$

14.



$\tan 14° = \frac{h}{AM}$; $\tan 10° = \frac{h}{BM}$

$AM = \frac{h}{\tan 14°}$ $\quad BM = \frac{h}{\tan 10°}$

$AB^2 + AM^2 = BM^2$ (Pyth. th.)

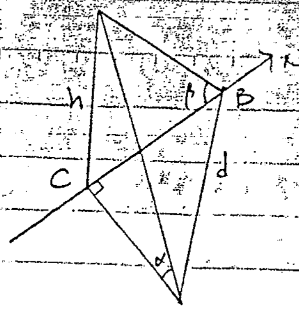
$7000^2 + \frac{h^2}{\tan^2 14°} = \frac{h^2}{\tan^2 10°}$

$7000^2 (\tan^2 14°)(\tan^2 10°) = h^2 (\tan^2 14° - \tan^2 10°)$

$h^2 = \frac{7000^2 \tan^2 14° \tan^2 10°}{\tan^2 14° - \tan^2 10°}$

$h = 1745.8m$

15.



$\cot\alpha = \frac{AC}{h}$; $\cot\beta = \frac{BC}{h}$

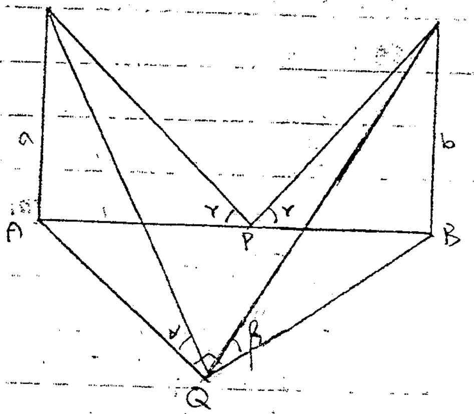
$AC = h\cot\alpha$; $BC = h\cot\beta$.

$\because BC^2 + AC^2 = AB^2$ (Pyth. th.)

$h^2\cot^2\beta + h^2\cot^2\alpha = d^2$

$\therefore \underline{d^2 = h^2(\cot^2\alpha + \cot^2\beta)}$

16, 米 diagram 二'''



$\cot\gamma = \frac{AP}{a}$; $\cot\gamma = \frac{PB}{b}$

$AP = a\cot\gamma$; $PB = b\cot\gamma$

$\cot\alpha = \frac{AQ}{a}$; $\cot\beta = \frac{BQ}{b}$

$AQ = a\cot\alpha$; $BQ = b\cot\beta$

$\because AQ^2 + BQ^2 = AB^2$ (Pyth. th.)

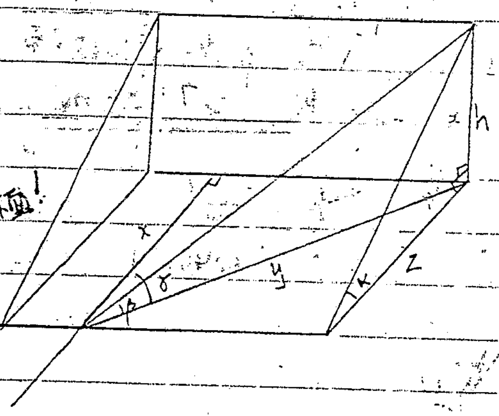
$a^2\cot^2\alpha + b^2\cot^2\beta = (a\cot\gamma + b\cot\gamma)^2$

$a^2\cot^2\alpha + b^2\cot^2\beta = a^2\cot^2\gamma + 2ab\cot^2\gamma + b^2\cot^2\gamma$

$a^2\cot^2\alpha + b^2\cot^2\beta = \cot^2\gamma\,(a^2 + 2ab + b^2)$

$\underline{a^2\cot^2\alpha + b^2\cot^2\beta = (a+b)^2\cot^2\gamma}$

17, 米 diagram... 米 bearing 係下面!



$\cos\gamma = \frac{x}{y}$

$\cot\alpha = \frac{z}{h}$

$\tan\beta = \frac{h}{y}$

$\because \cot\alpha \times \tan\beta = \frac{z}{h} \times \frac{h}{y}$

$= \frac{z}{y}$

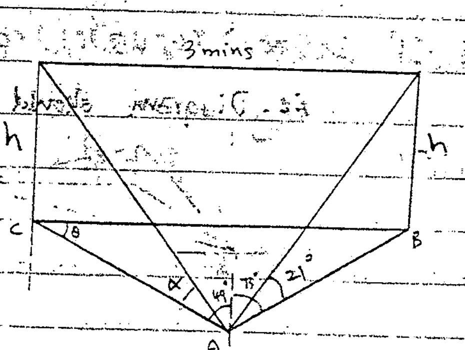
$\because z = x$

$\therefore \cot\alpha \times \tan\beta = \frac{x}{y}$

$= \cos\gamma$

$\therefore \underline{\cos\gamma = \cot\alpha\tan\beta}$

18.



240 Km/h

$\theta = 90° - 49° = 41°$

$\tan 21° = \frac{h}{AB}$

$AB = \frac{h}{\tan 21°}$

$BC = 240000 \times \frac{3}{60} = 12000$

$\because \frac{BC}{\sin 122°} = \frac{AB}{\sin 41°}$

$\frac{12000}{\sin 58°} = \frac{h}{\tan 21° \sin 41°}$

$\therefore \underline{h = 12000\tan 21°\sin 41°\operatorname{cosec} 58°}$

$\therefore \underline{h = 3564\text{m}}$

$\angle ABC = 17^\circ$ (remaining $\angle$)

$$\frac{AC}{\sin 17^\circ} = \frac{12000}{\sin 58^\circ}$$

$$\therefore\ AC = 12000 \sin 17^\circ \operatorname{cosec} 58^\circ$$

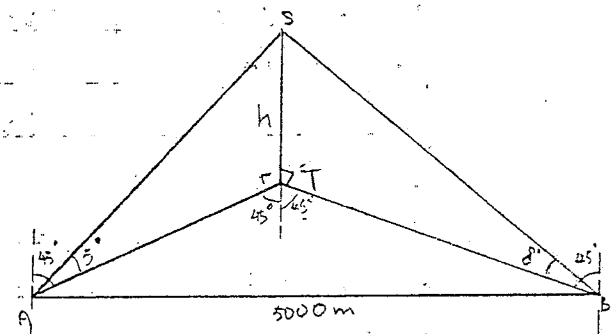
$$\therefore\ \tan \alpha = \frac{h}{AC}$$

$$\tan \alpha = \frac{12000 \sin 41^\circ \tan 21^\circ \operatorname{cosec} 58^\circ}{12000 \sin 17^\circ \operatorname{cosec} 58^\circ}$$

$$\therefore\ \underline{\underline{\tan \alpha = \sin 41^\circ \tan 21^\circ \operatorname{cosec} 17^\circ}}$$

$$\therefore\ \underline{\underline{\alpha = 40^\circ 44'}}$$

19.



$$\tan 5^\circ = \frac{h}{AT} \quad ; \quad \tan 8^\circ = \frac{h}{BT}$$

$$AT = \frac{h}{\tan 5^\circ} \qquad BT = \frac{h}{\tan 8^\circ}$$

$$AT^2 + BT^2 = 5000^2 \quad (\text{Pyth. } \dots)$$

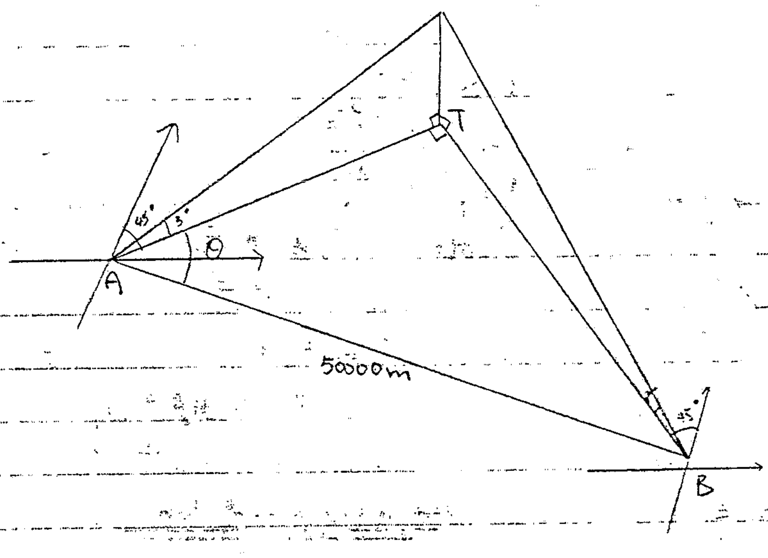
$$\frac{h^2}{\tan^2 5^\circ} + \frac{h^2}{\tan^2 8^\circ} = 5000^2$$

$$h^2(\tan^2 8^\circ + \tan^2 5^\circ) = 5000^2 \tan^2 8^\circ \tan^2 5^\circ$$

$$\therefore\ h^2 = \frac{5000^2 \tan^2 8 \tan^2 5^\circ}{\tan^2 8^\circ + \tan^2 5^\circ}$$

$$\underline{\underline{h = 371.4\,m}}$$

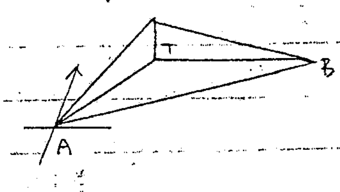
$$AT = \frac{371.4}{\tan 5^\circ} = 4245.121 \qquad BT = \frac{371.4}{\tan 8^\circ} = 2642.648$$



$$2642.648^2 = 5000^2 + 4245.121^2 - 2(5000)(4245.121)\cos\theta$$

$$\therefore\ \theta = 31^\circ 54'$$

i.e. Diagram should be ...



$$\therefore\ 45^\circ - \theta = 45^\circ - 31^\circ 54'$$

$$= \underline{\underline{13^\circ 6'}}$$