SOUTH SYDNEY HIGH SCHOOL
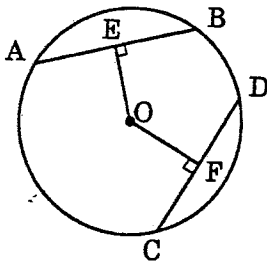## MATHEMATICS WORKSHEET
CIRCLE GEOMETRY

Name : _____   **Due :** _____

1.
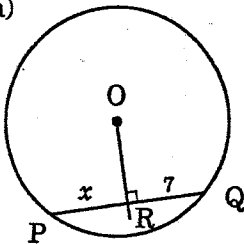

Prove that equal chords are equidistant from the centre, that is, given AB = CD, prove that OE = OF, where OE ⊥ AB and OF ⊥ CD.

Construction: Join OB and OD.

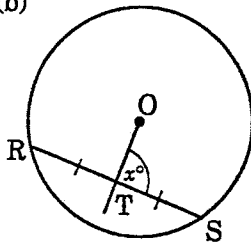*Hint*: Use congruent triangles.

2. Find the value of the pronumeral in the following questions. (All lengths are in cm.) Give adequate reasons.
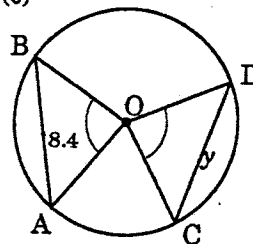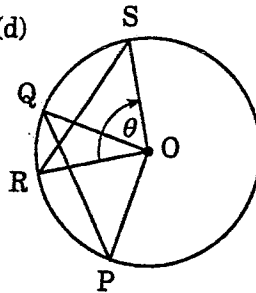
(a)

Given: RQ = 7 cm.

(b)

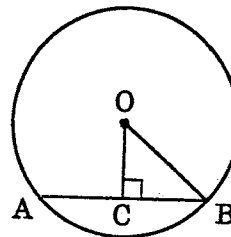Given: RT = TS.

(c)

Given: ∠AOB = ∠COD, AB = 8.4 cm.

(d)

Given: PQ = RS, ∠POQ = 110°, ∠ROS = θ°.

3. Use properties of the circle and Pythagoras' Theorem in the following questions:

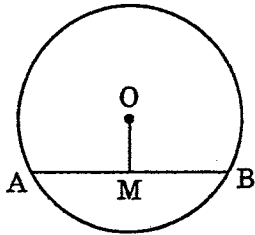**(a)**

AB is a 16 cm chord of a circle with radius 10 cm. Find the length OC.

Note: OC ⊥ AB.

(b)

Given that AM = MB, OM = 10 cm, and the circle has radius 26 cm, find the length of chord AB.
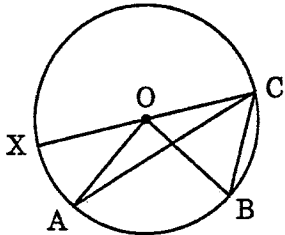
(c) A chord 15 cm long is 4 cm from the centre of a circle. Calculate the radius of the circle correct to one decimal place.

(d) Two parallel chords 32 cm long and 24 cm long are drawn in a circle of radius 20 cm as shown in the figure. Find the distance between the two chords if they are both on the same side of the centre.
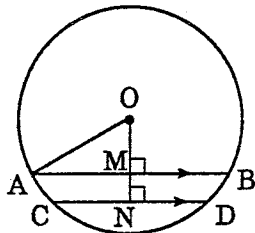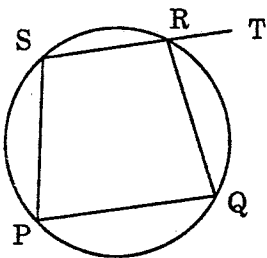

Given AB ‖ CD, AB = 32 cm, CD = 24 cm, OA = 20 cm. Find the length MN.

4.

Given that O is the centre of the circle, prove that ∠AOB = 2 × ∠ACB.
Construction: Join CO and produce it to X.
Let ∠OAC = $x°$, ∠OBC = $y°$ and remember that radii are equal.

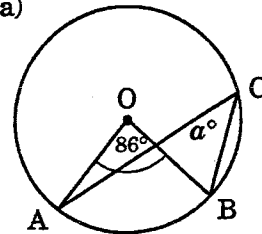**5.**

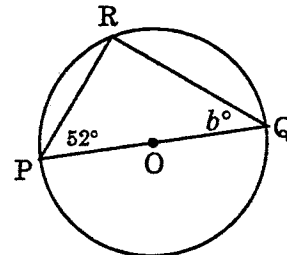Given that PQRS is a cyclic quadrilateral, prove that ∠SPQ = ∠QRT.

6. Find the value of the pronumeral(s) in each of the following diagrams, giving adequate reasons in each case.

(a)

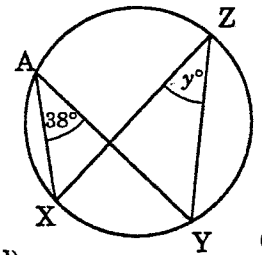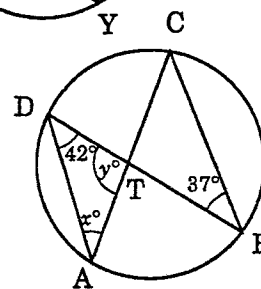Given ∠AOB = 86°, ∠ACB = $a°$.

(b)

Given PQ is a diameter, ∠RPQ = 52°, ∠RQP = $b°$.

(c)

Given ∠XAY = 38°, ∠XZY = $y°$.

(d)

Given ∠ADB = 42°, ∠DBC = 37°, ∠DAC = $x°$, ∠DTA = $y°$.

(e)

Given ∠DAB = 84°, ∠DCB = $g°$.

(f)

$\angle CBE = 114°$,
$\angle ADC = h°$.

(g)

AD ‖ BC, $\angle CBE = 114°$,
$\angle BAD = a°$, $\angle ADC = b°$, $\angle DCB = c°$.

(h)

AC is a diameter, $\angle BCO = 48°$,
$\angle OBC = y°$, $\angle ABO = x°$, $\angle BAC = z°$.

(i)

$\angle AOC = 138°$,
$\angle ABC = \theta°$.

(j)

$\angle PQR = 88°$,
$\angle POR = \phi°$.

(k)
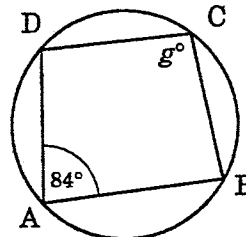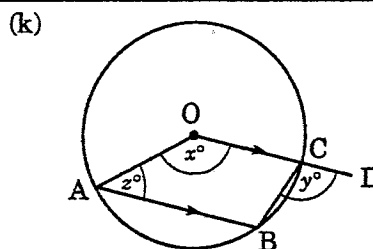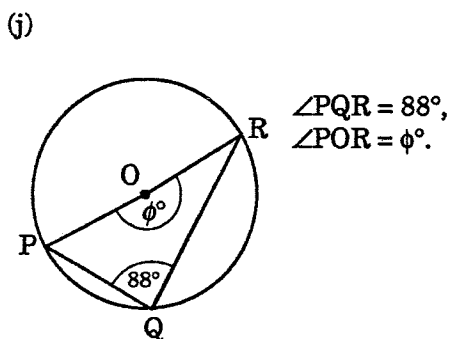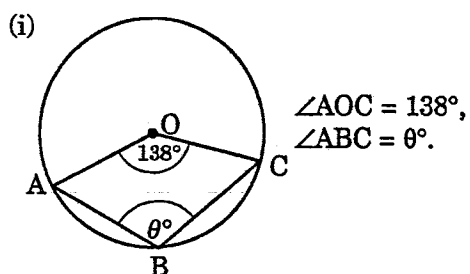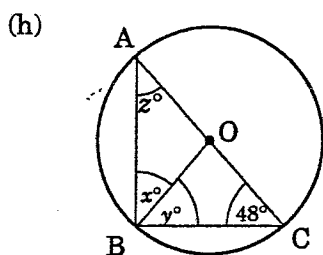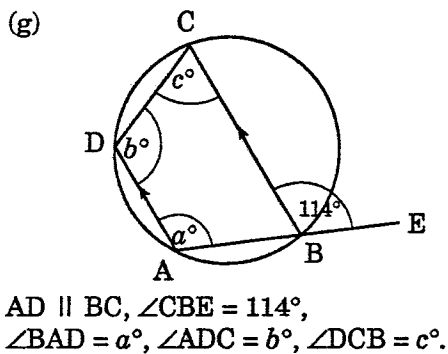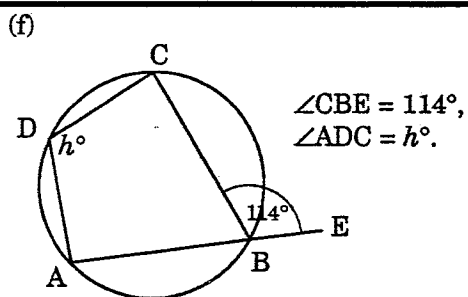
AB ‖ OD, $\angle ABC = 124°$, $\angle AOC = x°$,
$\angle BCD = y°$, $\angle BAO = z°$.

(l)

$\angle PSR = 48°$,
$\angle POR = a°$,
$\angle PQR = b°$.

(m)
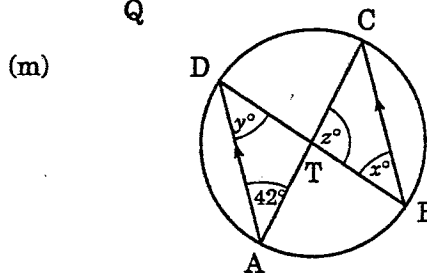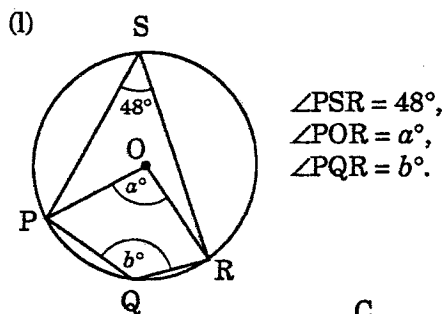
AB ‖ BC, $\angle DAC = 42°$, $\angle DBC = x°$,
$\angle ADB = y°$, $\angle CTB = z°$.

(n)
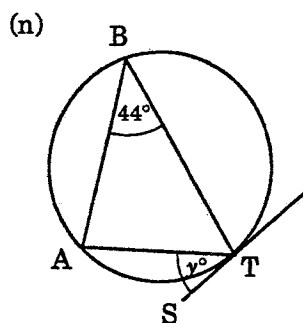
ST is a tangent at T, $\angle ABT = 44°$,
$\angle ATS = y°$.

(o)
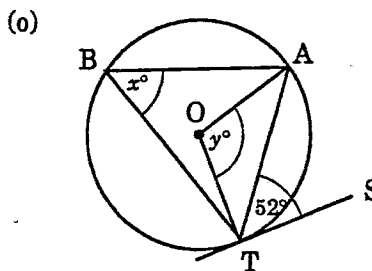
ST is a tangent at T, $\angle ATS = 52°$,
$\angle TBA = x°$, $\angle TOA = y°$.

(p)
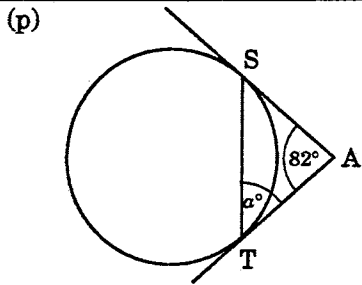
AS and AT are tangents at S and T,
∠SAT = 82°, ∠STA = a°.

(q)
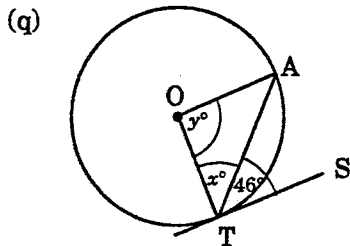
ST is a tangent at T, ∠ATS = 46°,
∠OTA = x°, ∠TOA = y°.

(r)
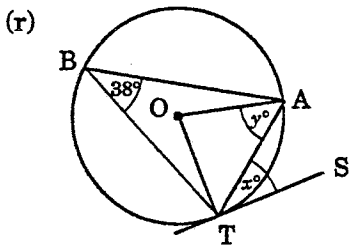
ST is a tangent at T, ∠TBA = 38°,
∠ATS = x°, ∠OAT = y°.

(s)
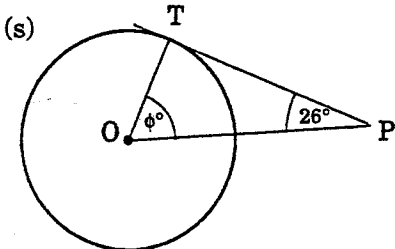
PT is a tangent at T, ∠OPT = 26°,
∠TOP = φ°.

(t)
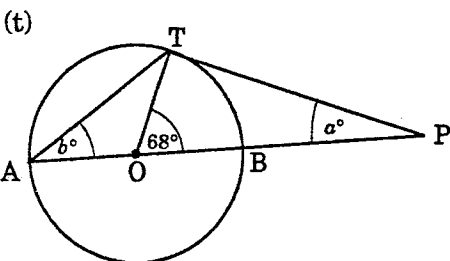
PT is a tangent at T, AB is a diam-
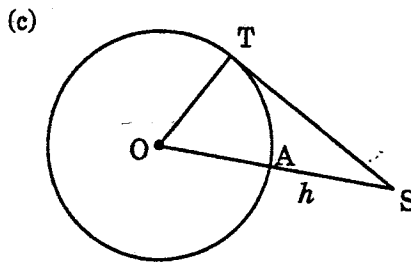eter, ∠TOB = 68°, ∠TPB = a°,
∠TAO = b°.

7. Find the value of h in each diagram. All
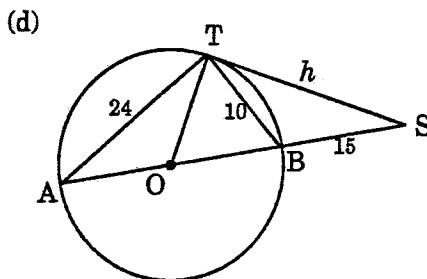   measurements are in cm.

(a)
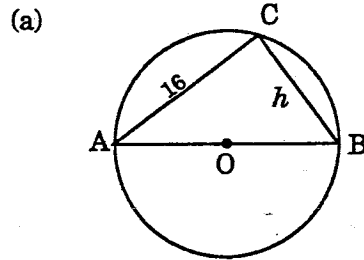
AB is the diameter of a circle of radius
10 cm, while AC = 16 cm, and BC = h cm.

(b)
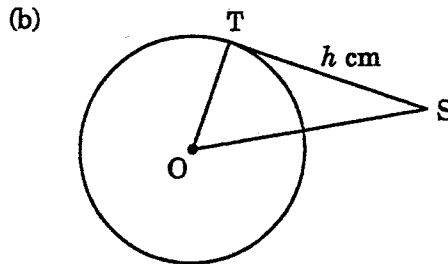
ST is a tangent at T to the circle of radius
15 cm. If OS is 25 cm, calculate h.

(c)

ST is a tangent at T to the circle of
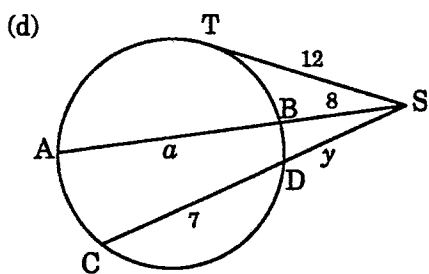radius 2.7 cm. Given that ST = 3.6 cm
and AS = h cm, calculate h.

(d)

ST is a tangent at T. AT = 24 cm,
BT = 10 cm, BS = 15 cm and ST = h cm.
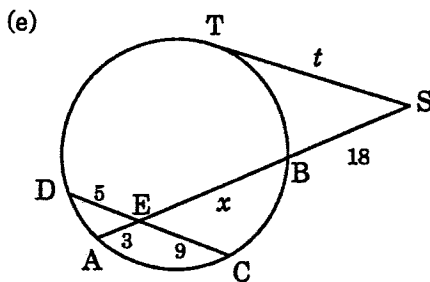Calculate h correct to one decimal place.

8. Calculate the value of the pronumeral
    in each diagram. All lengths are in cm.
    Answer correct to one decimal place
    where necessary.

(a)

AE = 5 cm,
EB = 8 cm,
ED = 4 cm,
EC = x cm.

(b)

AB = 7 cm,
BS = 8 cm,
TS = t cm.

(c)
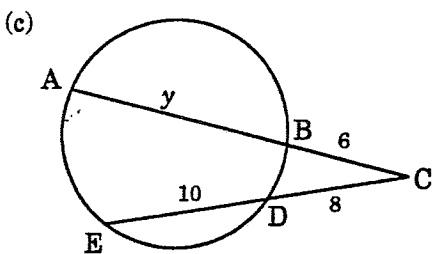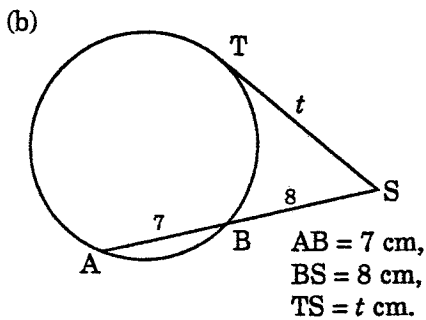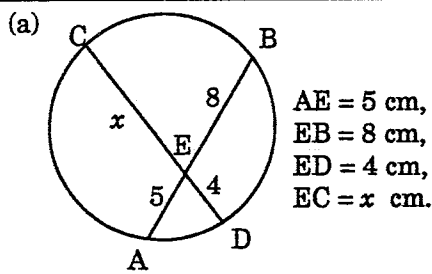
BC = 6 cm, DC = 8 cm, ED = 10 cm,
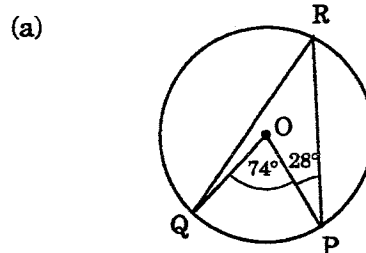AB = y cm.

(d)

ST = 12 cm, SB = 8 cm, AB = a cm,
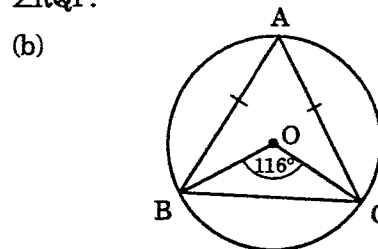CD = 7 cm, DS = y cm.

(e)

ST is a tangent at T. ED = 5 cm,
EA = 3 cm, EC = 9 cm, EB = x cm,
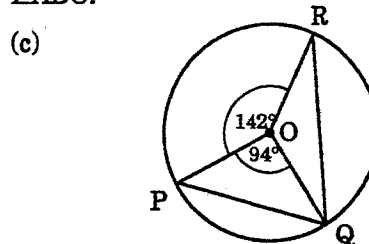BS = 18 cm, TS = t cm. (Leave t in
exact form.)

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9. Answer each part of this question with
a logical sequence of steps, giving
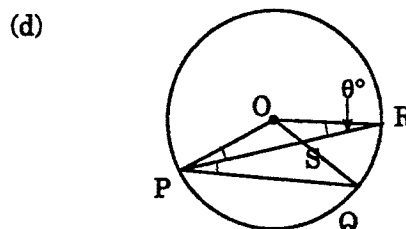adequate reasons. The point O is always
the centre of the circle.

(a)

Given ∠QOP = 74°, ∠OPR = 28°, find
∠RQP.

(b)

Given AB = AC, ∠BOC = 116°, find
∠ABO.

(c)

Given ∠POR = 142°, ∠POQ = 94°, find
∠ORQ.

(d)

Given ∠OPR = ∠QPR, ∠SRO = θ°, prove
∠ROS = 2θ°.

(e)

Given that ∠OXZ = θ°, and ∠XYZ = θ°,
prove that ∠XOZ = 90°.

(f)
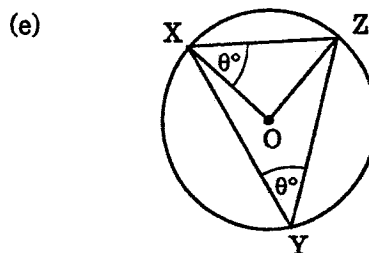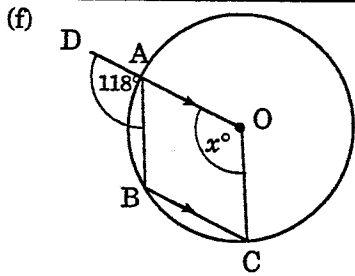
Given DO ∥ BC, ∠DAB = 118°, find x.

(g)
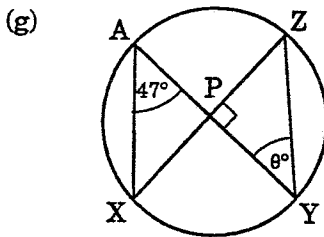
Given ∠XAY = 47°, ∠YPZ = 90°, find θ.

(h)
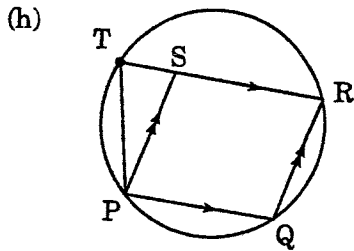
PQRS is a parallelogram, and PQRT is a cyclic quadrilateral. Prove that PT = PS. (Call ∠PQR = α°.)

(i)
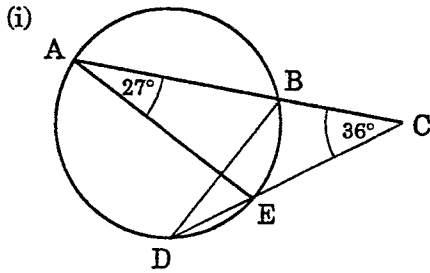
Given ∠BAE = 27°, ∠DCA = 36°, find ∠DBC.

(j)
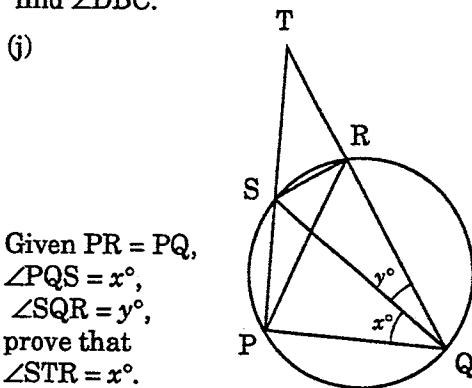
Given PR = PQ, ∠PQS = x°, ∠SQR = y°, prove that ∠STR = x°.

(k)
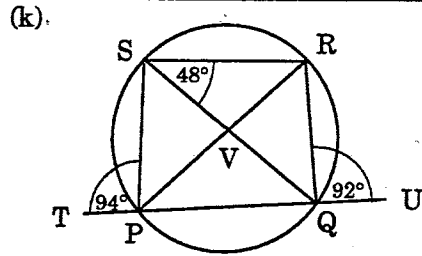
Given ∠TPS = 94°, ∠RQU = 92°, ∠QSR = 48°, find ∠SVR.

(l)
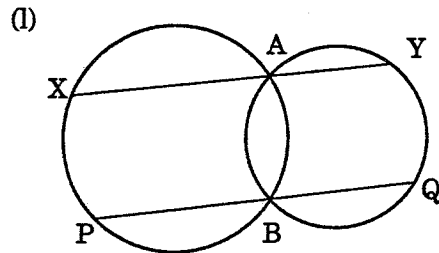
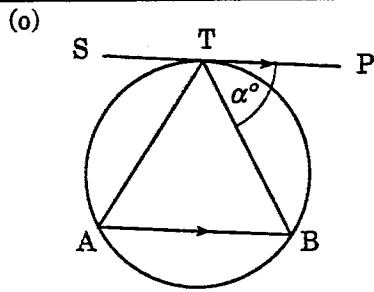Circles intersect at A and B. XY and PQ are straight lines. Prove that PX ∥ QY.

(m)
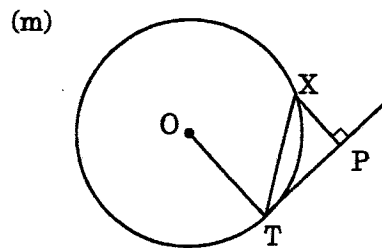
Given that PT is a tangent at T and XP ⊥ PT, prove that ∠OTX = ∠TXP.

(n)
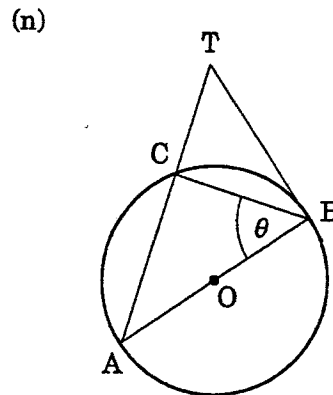
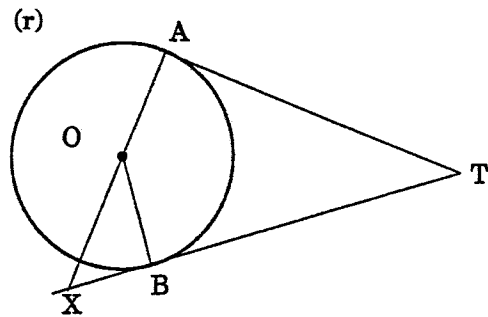Given that TB is a tangent at B and ∠ABC = θ°. AT is a straight line. Prove that ∠ATB = θ°.

(o)

Given that SP ‖ AB, PT is a tangent at T and $\angle PTB = \alpha°$, prove that $\triangle ABT$ is isosceles.

(p)
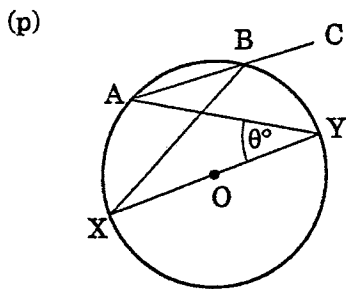
Given that AC is a straight line, XY is a diameter and $\angle XYA = \theta°$, prove that $\angle XYA + \angle YBC = 90°$.

(q)
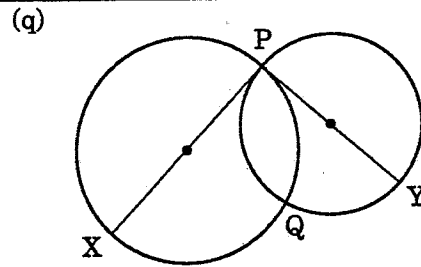
Two circles intersect at P and Q. PX and PY are the respective diameters. Prove that X, Q and Y are collinear.

(r)

TA and TB are tangents at A and B respectively. AX is a straight line. Prove that $\angle BOX = \angle ATB$ and that $\angle ATB = 2 \times \angle OAB$.