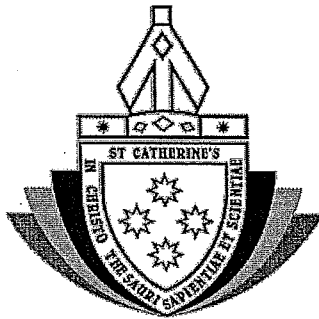# St. Catherine's School

Year 11 Extension 1 Mathematics
Preliminary Task #2
May 2009

Time allowed: 30 minutes + 5 minutes reading time
Total marks: 20 marks
Weighting: 15%

INSTRUCTIONS

- There are 3 questions of different values.
- Marks for each part of a question are indicated.
- All questions should be attempted in one booklet.
- All necessary working should be shown.
- Start each question on a new page.
- Approved scientific calculators and drawing templates may be used.
- Marks may be deducted for careless or badly arranged work.

Question 1 (10 marks)

a. Solve for x: $\dfrac{2x+5}{x+1} \leq 3$.   3

b. Find the acute angle between the straight lines
$2x + y - 4 = 0$ and $x - y - 2 = 0$, to the nearest degree.   3

c. Find the coordinates of the point R which divides the interval PQ,
where P is (2,-1) and Q is (-5,-3) externally in the ratio 7:4.   2

d. A point Q $\left(\dfrac{38}{13}, \dfrac{-1}{13}\right)$ divides the interval DC internally in the ratio 4:9. The
point D is (2,3). Find the coordinates of the point C.   2

Question 2 (5 marks) – Start a new page

Sketch the following graph highlighting the main features including the **limits** of the
function as x tends to infinity and **the x and y intercepts** where relevant. State clearly
the **domain** and **range**.
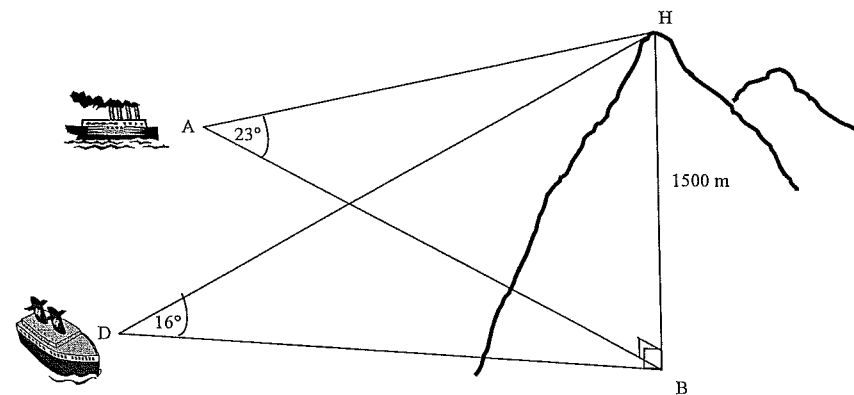
$$y = \dfrac{2}{x^2 - 5x + 6}$$ 5

Question 3 (5 marks) – Start a new page

David is in a life raft and Anne is in a cabin cruiser searching for him.
They are in contact by mobile telephone. David tells Anne that he can see
Mt Hope. From David's position the mountain has a bearing of 109°, the angle
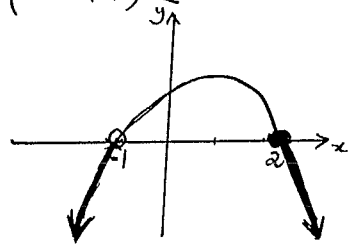of elevation to the top of the mountain is 16°.

Anne can also see Mt Hope. From her position it has a bearing of 139°, and the top of the
mountain has an angle of elevation of 23°.
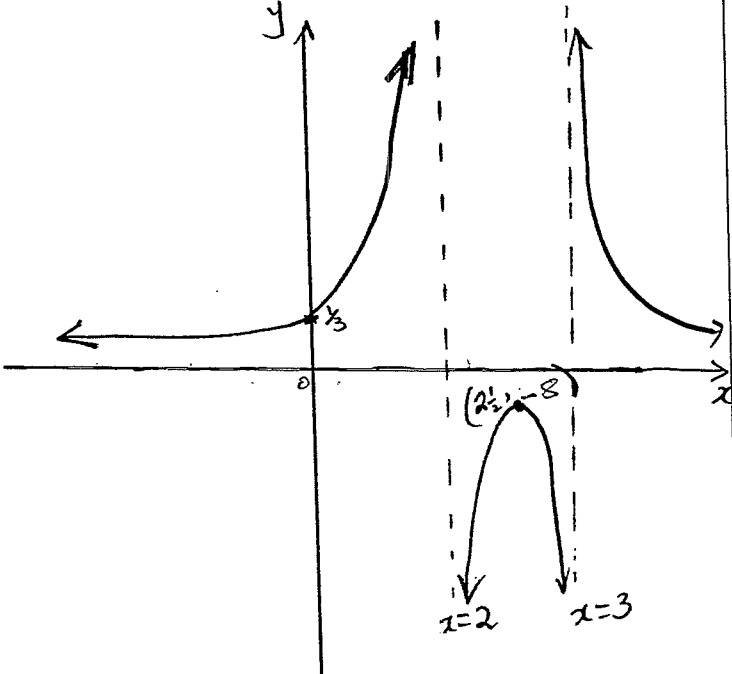The top of Mt Hope is 1500m above sea level.

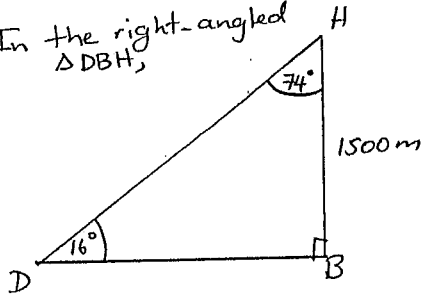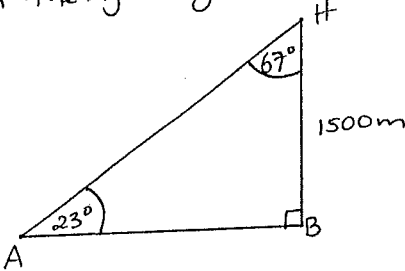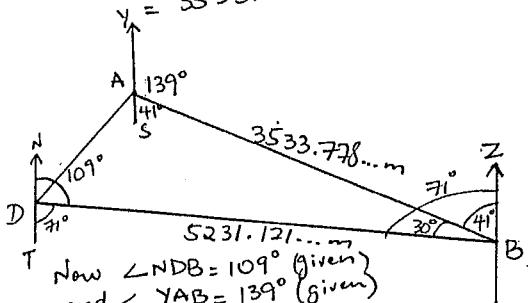**Find the distance and the bearing of the life raft from Anne's position.**   5

END OF PAPER

Qn	Solutions	Marks	Comments+Criteria
1	a. $\dfrac{2x+5}{x+1} \leq 3$		• 1 mark for arriving to this line *
	$\dfrac{(2x+5)(x+1)^2}{(x+1)} \leq 3(x+1)^2$		
	$(2x+5)(x+1) - 3(x+1)^2 \leq 0$ *		• 2 marks for sketch + solutions
	$(x+1)(2x+5 - 3(x+1)) \leq 0$		
	$(x+1)(2x+5-3x-3) \leq 0$	3	• ½ mark for $x \leq -1$
	$(x+1)(-x+2) \leq 0$		

$$x < -1 \quad \text{or} \quad x \geq 2$$

	b. $2x+y=4 \qquad x-y=2$		• 1 mark for correct value of m_1 & m_2
	$\quad y=4-2x \qquad y=x-2$		
	$\quad \therefore m_1 = -2 \qquad \therefore m_2 = 1$		
	$\tan\theta = \left| \dfrac{m_1 - m_2}{1+m_1 m_2} \right|$	3	• 1 mark for correct substitution and $\tan\theta = 3$
	$\quad = \left| \dfrac{-2-1}{1+-2\times1} \right|$		
	$\quad = \left| \dfrac{-3}{1-2} \right|$		• 1 mark for correct answer
	$\quad = \left| \dfrac{-3}{-1} \right|$		
	$\tan\theta = 3$		
	$\therefore \theta = 71°33'54.18''$		
	$\theta = 72° \text{ (nearest degree)}$		

Qn	Solutions	Marks	Comments+Criteria
1	c. $P\underset{\underset{x_1}{\downarrow}}{(2,}\underset{\underset{y_1}{\downarrow}}{-1)} \quad Q\underset{\underset{x_2}{\downarrow}}{(-5,}\underset{\underset{y_2}{\downarrow}}{-3)} \quad \begin{matrix}-7:4\\-m:n\end{matrix}$		
	$R\left(\dfrac{mx_2 + nx_1}{m+n}, \dfrac{my_2 + ny_1}{m+n}\right)$		1 mark for correct formula and $\ominus m:n$
	$R\left(\dfrac{-7\times-5 + 4\times2}{-7+4}, \dfrac{-7\times-3 + 4\times-1}{-7+4}\right)$	2	
	$R\left(\dfrac{35+8}{-3}, \dfrac{21-4}{-3}\right)$		1 mark for correct answer
	$R\left(\dfrac{43}{-3}, \dfrac{17}{-3}\right)$		
	d. $\dfrac{38}{13} = \dfrac{mx_2 + nx_1}{m+n} \qquad \begin{matrix}m:n\\4:9\end{matrix}$		1 mark for correct expressions
	$\dfrac{38}{13} = \dfrac{4x_2 + 9\times2}{4+9} \qquad \dfrac{-1}{13} = \dfrac{4y_2 + 9\times3}{13}$		
	$\dfrac{38}{13} = \dfrac{4x_2 + 18}{13} \qquad -1 = 4y + 27$	2	1 mark for correct answer
	$\qquad\qquad\qquad\qquad 4y = -28$		
	$4x + 18 = 38 \qquad\qquad y = -7$		
	$4x = 38-18$		
	$4x = 20$		
	$x = 5$		
	$C(5, -7)$		

Qn	Solutions	Marks	Comments+Criteria
2.	$y = \dfrac{2}{x^2 - 5x + 6}$		

$\bullet$ x-int $\rightarrow$ let y=0

$$0 = \frac{2}{x^2 - 5x + 6}$$

$\therefore$ no x-int.

$\bullet$ y-int $\rightarrow$ let x=0

$$y = \frac{2}{0^2 - 5(0) + 6}$$

$$y = \frac{2}{6}$$

$$y = \frac{1}{3}$$

$\bullet$ vertical asymptote

$$y = \frac{2}{(x-3)(x-2)}$$

$$x = 3 \; , \; x = 2$$

$\bullet$ horizontal asymptote

$$\lim_{x \to \infty} \frac{\frac{2}{x^2}}{\frac{x^2}{x^2} - \frac{5x}{x^2} + \frac{6}{x^2}}$$

$\therefore y = 0$

D: For all real x, $x \neq 2$, $x \neq 3$

R: $y > 0$, $y \leqslant -8$

Marks: 5

Comments+Criteria:

1 mark for x-&y intercepts

1 mark for horizontal + vertical asymptote

1 mark for D & R

2 mark for sketch

Qn	Solutions	Marks	Comments+Criteria
3	In the right-angled $\triangle DBH$, $\tan 74° = \dfrac{DB}{1500}$ $\therefore DB = 1500 \tan 74°$ $= 5231.121\ldots$ In the right-angled $\triangle ABH$, $\tan 67° = \dfrac{AB}{1500}$ $AB = 1500 \tan 67°$ $= 3533.778\ldots$ Now $\angle NDB = 109°$ (given) and $\angle YAB = 139°$ (given) $\therefore \angle SAB = 180° - 139°$ $= 41°$ and $\angle TDB = 180° - 109°$ $= 71°$		✗ means 0.5 mark.

Qn	Solutions	Marks	Comments+Criteria
3	Also, $\angle SAB = \angle ZBA = 41°$ (alt. $\angle$s, $SY \parallel BZ$) and $\angle TDB = \angle ZBD = 71°$ (alt. $\angle$s, $TN \parallel BZ$) $\therefore \angle DBA = 71° - 41°$ $= 30°$ In $\triangle DAB$, $AD^2 = AB^2 + DB^2 - 2(AB)(DB)\cos 30°$ $= (1500 \tan 67°)^2 + (1500 \tan 74°)^2 -$ $\qquad 2 \times 1500 \tan 67° \times 1500 \tan 74° \cos 30°$ $= 7834\,182.089\ldots$ $\therefore AD = 2798.960\ldots$ m $\therefore$ The distance of the life raft from Anna's position is approximately 2799 m. In $\triangle DAB$, $DB^2 = AD^2 + AB^2 - 2(AD)(AB)\cos \angle DAB$ $1500 \tan 74° = (2798.960\ldots)^2 + (1500 \tan 67°)^2 -$ $\qquad 2 \times 2798.960\ldots \times 1500 \tan 67° \cos \angle DAB$ $\therefore \cos \angle DAB = \dfrac{(2798.960\ldots)^2 + (1500 \tan 67°)^2 - 1500 \tan 74°}{2 \times 2798.960\ldots \times 1500 \tan 67°}$ $= -0.35602\ldots$ $\therefore \angle DAB = 110.856\ldots°$ $\div 111°$ $\therefore$ The bearing of D (life raft) from A (Anne's position is $139° + 111°$ $= 250°$	3 2	