

# St Catherine's School

Year: 12 Extension 1  
Subject: Mathematics  
Time allowed: 55 minutes  
Date: February 2006

Student number \_\_\_\_\_

Teacher's Name: \_\_\_\_\_

**Directions to candidates:**

- All questions are to be attempted.
- Marks may be deducted for careless or badly arranged work
- All necessary **working** must be shown
- Approved calculators may be used

Marks:

Q 1	7 / 8
Q 2	7 / 17
Q 3	7 / 8
Q 4/5	8 / 8
Total	29 / 31

**Question 1 (8 marks)**

The tangent at the point P  $(2ap, ap^2)$  on the parabola  $x^2 = 4ay$  meets the axis of the parabola at T.

S is the focus of the parabola.

- i) Sketch the parabola showing P, T and S 1
- ii) Find the equation of the tangent at P 2
- iii) Show that the co-ordinates of T are  $(0, -ap^2)$  1
- iv) Write down the co-ordinates of S 1
- v) Show that  $SP=ST$  3

**Question 2 (7 marks)**

P  $(2ap, ap^2)$  and Q  $(2aq, aq^2)$  are points on the parabola  $x^2 = 4ay$  such that OP is perpendicular to OQ where O is the vertex of the parabola.

- i) Show that  $pq = -4$  2
- ii) Find the co-ordinates of M, the midpoint of chord PQ 1
- iii) Show that the locus of M has Cartesian equation  $x^2 = 2a(y - 4a)$  and describe this curve. 3
- iv) Describe the curve. 1

**Question 3 (8 marks)**

- ii) When the polynomial  $P(x)$  is divided by  $(x-4)(x-3)$ , the remainder is  $2x + 3$ . Find the remainder when  $P(x)$  is divided by  $x-4$

2

i)  $Q(x) = x^3 + 6x^2 + 3x - 10$

- a) Show  $Q(x)$  has a factor  $(x-1)$   
b) Hence express  $Q(x)$  in fully factored form.

1

2

- iii) A polynomial  $P(x)$  of degree 3 has roots at  $x = 1, 4$  and  $6$ , and  $P(0) = 12$ .

- a) Find  $P(x)$   
b) Sketch  $P(x)$

2

1

**Question 4 (4 marks)**

Show that the polynomial  $P(x) = x^4 + x - 5$  has a zero between 1 and 2.

1

Using one application of Newton's method with a first approximation to the zero of 1.5, find a more accurate value for the zero.

3

**Question 5 (4 marks)**

For the polynomial

$$P(x) = 3x^3 + 12x^2 - 9x - 4$$

with roots  $\alpha, \beta, \gamma$ , find the value of

i)  $\alpha + \beta + \gamma$

1

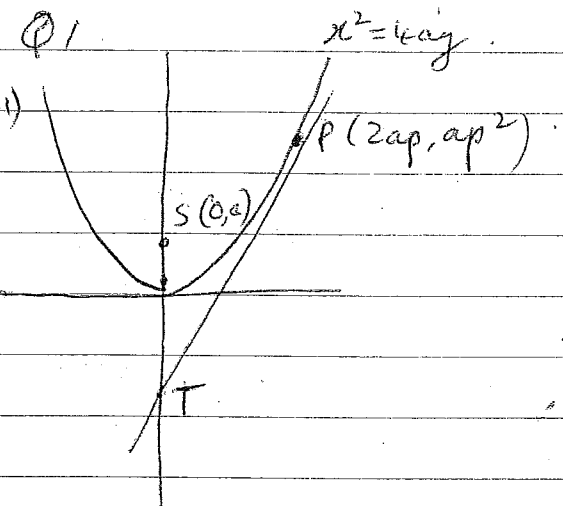
ii)  $\alpha\beta + \alpha\gamma + \beta\gamma$

1

iii)  $\alpha^2 + \beta^2 + \gamma^2$

2

# Solutions Jib Ext 1 Maths



(i)  $x^2 = 4ay$   
 $y = \frac{x^2}{4a}$   
 $\frac{dy}{dx} = \frac{2x}{4a} = \frac{x}{2a}$

at  $x = 2ap$ ,  $\frac{dy}{dx} = \frac{2ap}{2a} = p$

$\therefore$  eq of tangent

$$y - ap^2 = p(x - 2ap)$$

$$y = px - ap^2$$

(iii)  $T(0, \quad)$

at  $x = 0$  on tangent

$$y = p \times 0 - ap^2$$

$$= -ap^2$$

$\therefore T(0, -ap^2)$

(iv)  $S(0, a)$

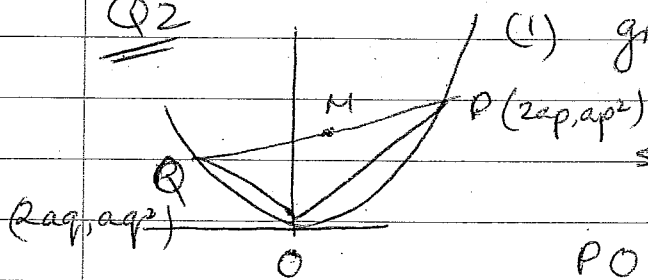
(v)  $SP = \sqrt{(2ap - 0)^2 + (ap^2 - a)^2}$   
 $= \sqrt{4a^2p^2 + a^2(p^2 - 1)^2}$   
 $= a(p^2 + 1)$

From diagram,

$$ST = a + ap^2$$

$\therefore ST = SP$

Q2



(i)  $\text{grad } OP = \frac{ap^2 - 0}{2ap - 0} = \frac{p}{2}$

$\text{sum grad } OQ = \frac{q}{2}$

$PO \perp OQ \therefore \frac{p}{2} \times \frac{q}{2} = -1$

$\therefore pq = -4$

(ii)  $P1 = \left( \frac{2ap + 2aq}{2}, \frac{ap^2 + aq^2}{2} \right)$   
 $= \left( a(p+q), \frac{a}{2}(p^2 + q^2) \right)$

(iii)  $x = a(p+q)$

$y = \frac{a}{2}(p^2 + q^2) \therefore p^2 + q^2 = \frac{2y}{a}$

$(p+q) = \frac{x}{a}$

$(p^2 + q^2) = \frac{x^2}{a^2}$

$p^2 + q^2 + 2pq = \frac{x^2}{a^2}$

$\frac{2y}{a} + -8 = \frac{x^2}{a^2}$

$2ay - 8a^2 = x^2$

$\therefore x^2 = 2a(y - 4a)$

focal length  $\frac{a}{2}$

$$Q3 \text{ i) } P(x) = (x-4)(x-3)Q(x) + 2x+3$$

$$P(4) = 0 \times 1 \times Q(x) + 8+3$$

$$\therefore P(4) = 11$$

Remainder is 11.

$$\text{ii) a) } Q(x) = x^3 + 6x^2 + 3x - 10$$

$$Q(1) = 1 + 6 + 3 - 10$$

$$= 0$$

$\therefore x-1$  is a factor.

$$\text{b) } \begin{array}{r} x^2 + 7x + 10 \\ x-1 \overline{) x^3 + 6x^2 + 3x - 10} \\ \underline{x^3 - x^2} \phantom{+ 3x - 10} \\ 7x^2 + 3x \phantom{- 10} \\ \underline{7x^2 - 7x} \phantom{- 10} \\ 10x - 10 \end{array}$$

$$\therefore P(x) = (x-1)(x^2 + 7x + 10)$$

$$= (x-1)(x+2)(x+5)$$

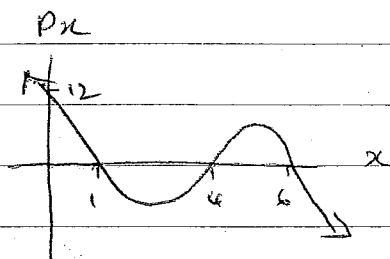
iii)

$$P(x) = k(x-1)(x-4)(x-6)$$

$$P(0) = k \times -1 \times -4 \times -6 = 12$$

$$-24k = 12 \quad \therefore k = -\frac{1}{2}$$

$$P(x) = -\frac{1}{2}(x-1)(x-4)(x-6)$$



$$Q4 \text{ } P(x) = x^4 + x - 5$$

$$P(1) = 1 + 1 - 5 < 0$$

$$P(2) = 16 + 2 - 5 > 0$$

change of sign  $\therefore$  root betw 1 & 2

$$x_2 = x_1 - \frac{P(x_1)}{P'(x_1)} = 1.5 - \frac{P(1.5)}{P'(1.5)}$$

$$= 1.39$$

$$Q5 \quad \alpha + \beta + \gamma = -\frac{b}{a} = -\frac{12}{3} = -4$$

$$\alpha\gamma + \beta\gamma + \alpha\beta = \frac{c}{a} = \frac{-9}{3} = -3$$

$$\begin{aligned} \alpha^2 + \beta^2 + \gamma^2 &= (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma) \\ &= (-4)^2 - 2 \times -3 = \underline{\underline{22}} \end{aligned}$$