St Catherine's School

Year: 12 Extension 1
Subject: Mathematics
Time allowed: 55 minutes
Date: February 2006

Student number______________________________

Teacher's Name: ____________________

Directions to candidates:

- All questions are to be attempted.
- Marks may be deducted for careless or badly arranged work
- All necessary **working** must be shown
- Approved calculators may be used

Marks:

Q 1	/8
Q 2	/7
Q 3	/8
Q 4/5	/8
Total	/31

Question 1 (8 marks)

The tangent at the point P $(2ap, ap^2)$ on the parabola $x^2 = 4ay$ meets the axis of the parabola at T.
S is the focus of the parabola.

i) Sketch the parabola showing P,T and S 1

ii) Find the equation of the tangent at P 2

iii) Show that the co-ordinates of T are $(0, -ap^2)$ 1

iv) Write down the co-ordinates of S 1

v) Show that SP=ST 3

Question 2 (7 marks)

P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ are points
on the parabola $x^2 = 4ay$ such that OP is perpendicular to OQ
where O is the vertex of the parabola.

i) Show that $pq = -4$ 2

ii) Find the co-ordinates of M, the midpoint of chord PQ 1

1

iii) Show that the locus of M has Cartesian equation
$x^2 = 2a(y - 4a)$ and describe this curve. 3

iv) Describe the curve. 1

Question 3 (8 marks)

ii) When the polynomial $P(x)$ is divided by $(x-4)(x-3)$, the remainder is $2x + 3$. Find the remainder when $P(x)$ is divided by $x-4$ 2

i) $Q(x) = x^3 + 6x^2 + 3x - 10$

a) Show $Q(x)$ has a factor $(x-1)$ 1

b) Hence express $Q(x)$ in fully factored form. 2

iii) A polynomial P(x) of degree 3 has roots at x = 1, 4 and 6, and P(0) = 12.

a) Find P(x) 2

b) Sketch P(x) 1

Question 4 (4 marks)

Show that the polynomial $P(x) = x^4 + x - 5$ has a zero between 1 and 2. 1

Using one application of Newton's method with a first approximation to the zero of 1.5, find a more accurate value for the zero. 3

Question 5 (4 marks)

For the polynomial

$P(x) = 3x^3 + 12x^2 - 9x - 4$

with roots α, β, γ, find the value of

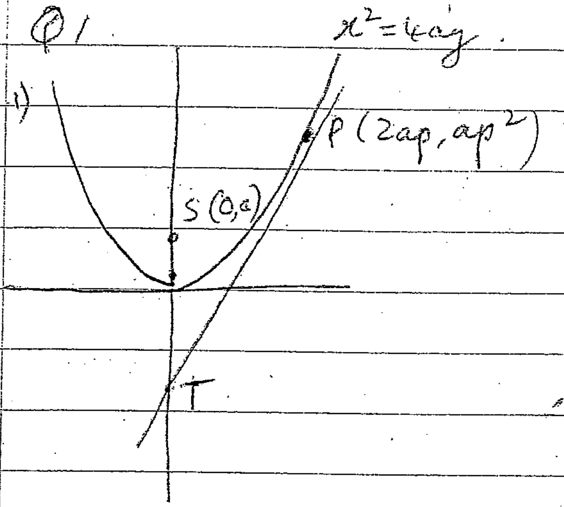
i) $\alpha+\beta+\gamma$ 1

ii) $\alpha\beta+\alpha\gamma+\beta\gamma$ 1

iii) $\alpha^2+\beta^2+\gamma^2$ 2

Solutions Feb '06 Ext 1 Maths

Q1



(ii) $x^2 = 4ay$

$$y = \frac{x^2}{4a}$$

$$\frac{dy}{dx} = \frac{2x}{4a} = \frac{x}{2a}$$

at $x = 2ap$, $\frac{dy}{dx} = \frac{2ap}{2a} = p$

$\therefore$ eq of tangent

$$y - ap^2 = p(x - 2ap)$$

$$y = px - ap^2$$

(iii) $T(0,\ \ )$

at $x=0$ on tangent

$$y = p \times 0 - ap^2$$

$$= -ap^2$$

$\therefore T(0, -ap^2)$

(iv) $S(0,a)$

(v) $SP = \sqrt{(2ap-0)^2 + (ap^2-a)}$

$$= \sqrt{a^2p^4 + 2a^2p^2 + a^2}$$

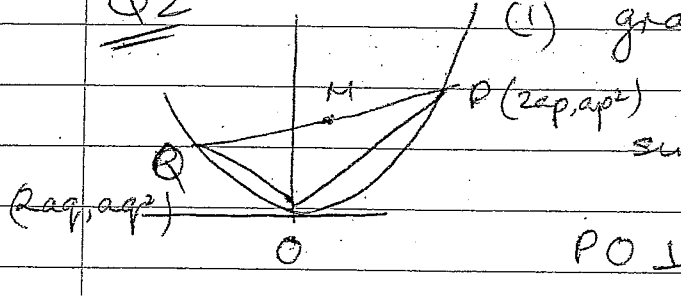
$$= a(p^2+1)$$

From diagram,

$$ST = a + ap^2$$

$\therefore ST = SP$

Q2



(i) grad $OP = \frac{ap^2-0}{2ap-0} = \frac{p}{2}$

sim grad $OQ = \frac{q}{2}$

$PO \perp OQ \therefore \frac{p}{2} \times \frac{q}{2} = -1$

$\therefore pq = -4$

ii) $M = \left(\frac{2ap+2aq}{2}, \frac{ap^2+aq^2}{2}\right)$

$$= \left(a(p+q), \frac{a}{2}(p^2+q^2)\right)$$

(iii) $x = a(p+q)$

$(p+q) = \frac{x}{a}$

$(p+q)^2 = \frac{x^2}{a^2}$

$p^2+q^2+2pq = \frac{x^2}{a}$

$y = \frac{a}{2}(p^2+q^2) \therefore p^2+q^2 = \frac{2y}{a}$

$\therefore \frac{2y}{a} \cancel{+} -8 = \frac{x^2}{a^2}$

$2ay - 8a^2 = x^2$

$\therefore x^2 = 2a(y-4a)$

focal length $\frac{a}{2}$

Q3 i) $P(x) = (x-4)(x-3)\,Q(x) + 2x+3$

$P(4) = 0 \times 1 \times Q(x) + 8+3$

$\therefore P(4) = 11$

Remainder is 11.

ii) a) $Q(x) = x^3 + 6x^2 + 3x - 10$

$Q(1) = 1 + 6 + 3 - 10$

$= 0 \quad \therefore x-1$ is a factor.

b)
$$\begin{array}{r} x^2+7x+10 \\ x-1 \overline{) x^3+6x^2+3x-10} \\ x^3-x^2 \\ \hline 7x^2+3x \\ 7x^2-7x \\ \hline 10x-10 \end{array}$$

$\therefore P(x) = (x-1)(x^2+7x+10)$

$= (x-1)(x+2)(x+5)$

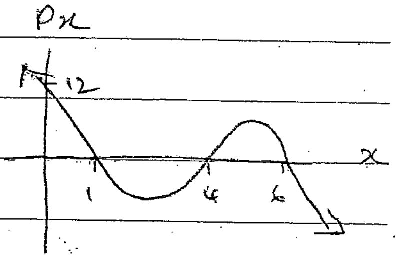
iii)

$P(x) = k(x-1)(x-4)(x-6)$

$P(0) = k \times -1 \times -4 \times -6 = 12$

$-24k = 12 \quad \therefore k = -\frac{1}{2}$

$P(x) = -\frac{1}{2}(x-1)(x-4)(x-6)$



Q4 $P(x) = x^4 + x - 5$

$P(1) = 1+1-5 < 0$

$P(2) = 16+2-5 > 0$ change of sign $\therefore$ root betw 1&2

$x_2 = x_1 - \frac{P(x_1)}{P'x_1} = 1.5 - \frac{P(1.5)}{P'(1.5)}$

$\doteqdot 1.39$

Q5 $\alpha+\beta+\gamma = \frac{-b}{a} = \frac{-12}{3} = \underline{\underline{-4}}$

$\alpha\gamma + \beta\gamma + \alpha\beta = \frac{c}{a} = \frac{-9}{3} = \underline{\underline{-3}}$

$\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$

$= (-4)^2 - 2 \times -3 = \underline{\underline{22}}$