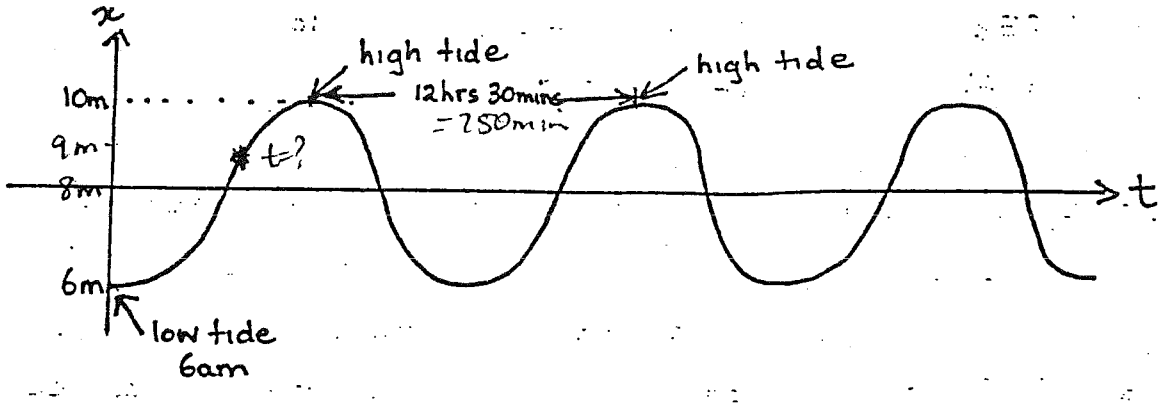
SHM.. TIDE QUESTIONS

Kaye

1. The rise and fall of the tide approximate S.H.M. The interval between successive high tides in a harbour is 12hrs and 30mins In the entrance the depth of water at high tide is 10m and at low tide is 6m. If low tide occurs at 6am, at what time will the depth of water in the harbour entrance first reach 9m.



amplitude of motion

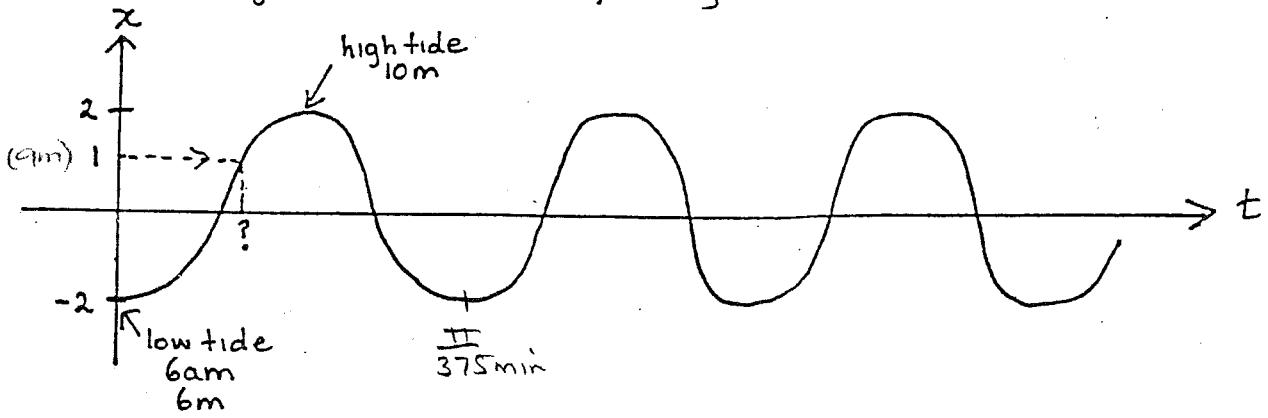
$A = 2m$

period of motion $= \frac{2\pi}{n}$

$= \frac{2\pi}{750}$

$= \frac{\pi}{375}$ min.

redraw the graph with everything centred



equation of motion is $x = -a\cos nt$

now need time when depth is 9m - on graph this is at $x = 1$

$$1 = -2\cos\left(\frac{\pi}{375}\right)t$$

$$\cos\left(\frac{\pi}{375}\right)t = -\frac{1}{2},$$

$t = 250$

$t = 4$hrs 10min

6am + 4hrs 10 min $= 10:10$ am

2. The deck of a ship was 2.4m below the level of a wharf at low tide and 0.6m above wharf level at high tide. Low tide was at 8.30am and high tide at 2.35pm. Find the first time after low tide that the deck was level with the wharf. [12.47pm]

3. The deck of a ship was 3.0m below the level of a wharf at low tide and 2.6m above wharf level at high tide. Low tide was at 7.10am and high tide at 1.25pm. Find the first time after low tide that the deck was level with the wharf [10.26am]

4. On a particular Sunday the low tide mark on a wharf was 4.2m below the edge of the wharf. Low tide occurred at 9.00am. The high tide mark was 1.4m below the edge of the wharf and high tide occurred at 3.05pm. A boat, tied to the wharf, had a deck which was 2.6m above water level. Assuming that the rise and fall of the tide is S.H.M, at what times during the afternoon was the deck exactly level with the wharf [12.19pm, 5.51pm]

5. On a certain day high water for a harbour occurs at 5am and low water at 11.20am, the corresponding depths being 30m and 10m. If the tidal motion is assumed to be S.H.M, prove that to the nearest minute, the latest time before noon that a ship, drawing 25m, can enter the harbour is 7.06am

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. $a = 1.5$

$n = \frac{\pi}{365}$

qun of motion $x = -1.5\cos\frac{\pi}{365}t$

eed t when $x = 0.9$

3. $a = 2.8$

$n = \frac{\pi}{375}$

equn of motion $x = -2.8\cos\frac{\pi}{375}t$

need t when $x = 0.2$

. $a = 1.4$

$n = \frac{\pi}{365}$

qun of motion $x = -1.4\cos\frac{\pi}{365}t$

eed t when $x = 0.2$ $(n = 0, 1)$

5. $a = 10$

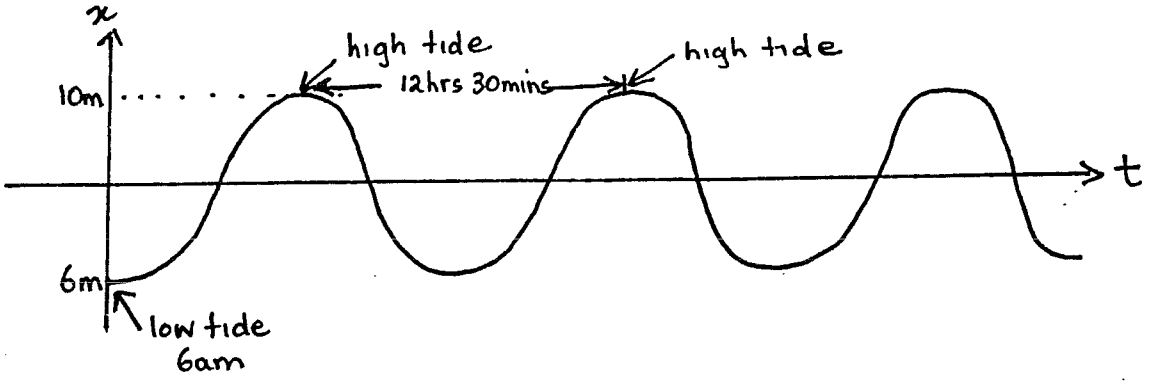
$n = \frac{\pi}{380}$

equn of motion $x = 10\cos\frac{\pi}{180}t$

need t when $x = 5$.

S.H.M – Tide Questions – SOLUTIONS

1. The rise and fall of the tide approximate S.H.M. The interval between successive high tides in a harbour is 12hrs and 30mins. In the entrance the depth of water at high tide is 10m and at low tide is 6m. If low tide occurs at 6am, at what time will the depth of water in the harbour entrance first reach 9m.



amplitude of motion $2a = 4 \quad (10-6)$

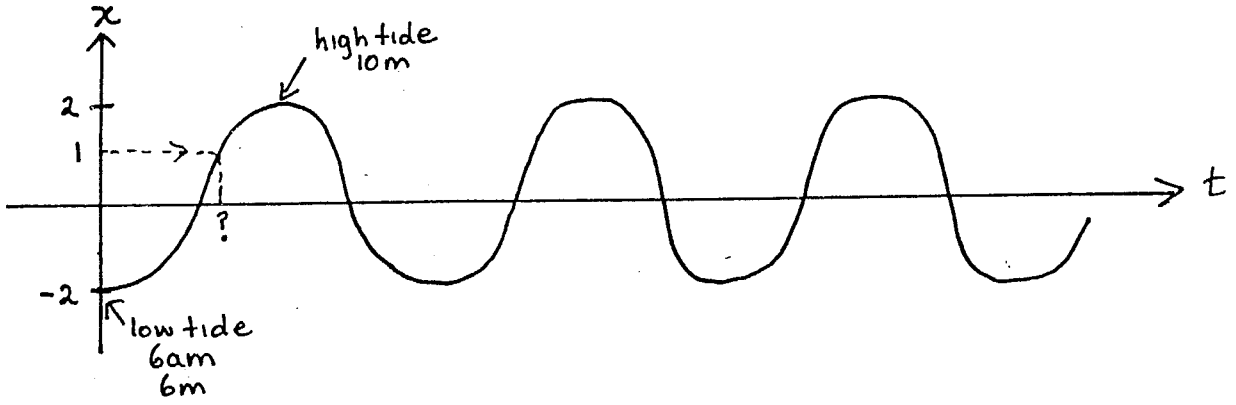
$a = 2$

period of motion $= 750$ mins

ie $\frac{2\pi}{n} = 750$

$n = \frac{\pi}{375}$

redraw the graph with everything centred



equation of motion is $x = -2\cos\frac{\pi}{375}t$ $\quad (t=0 \text{ at 6am} \quad x=-2 \text{ 6m} \quad \text{ie cos curve})$

now need time when depth is 9m – on graph this is at $x = 1$

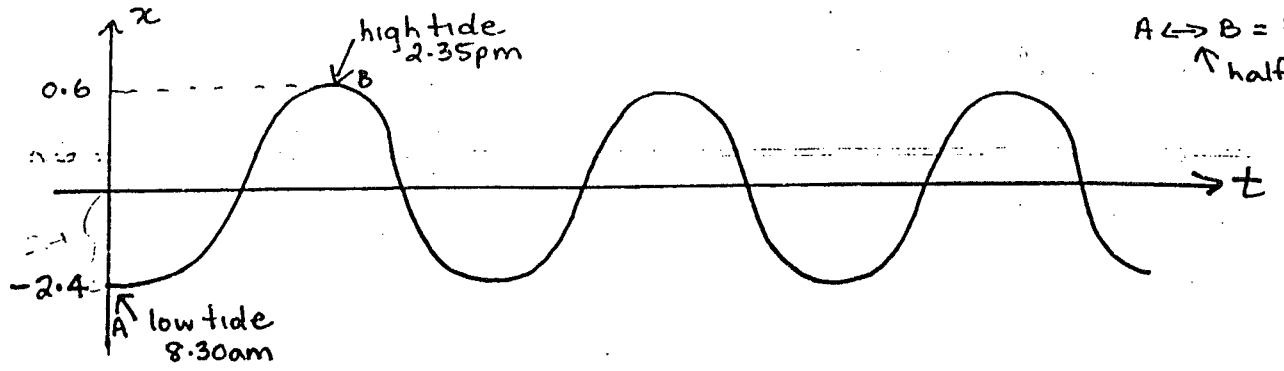
ie $1 = -2\cos\frac{\pi}{375}t$

ie $t = \frac{375}{\pi}\cos^{-1}\left(-\frac{1}{2}\right)$

$t = 250$ mins

$\therefore$ depth will first reach 9m at 6am + 250 mins = 10.10 am

2.



amplitude of motion $2a = 0.6 + 2.4$

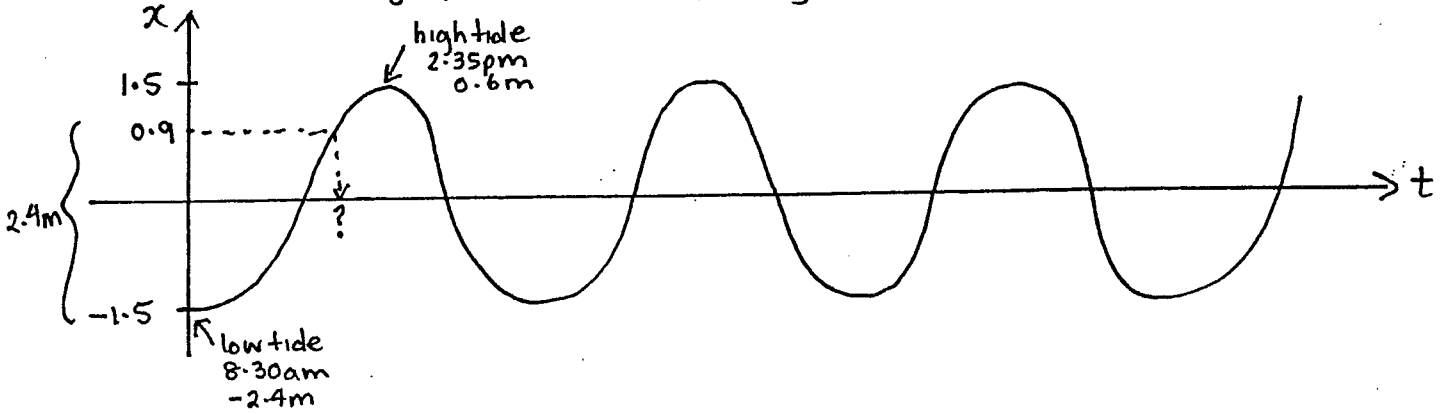
$a = 1.5$

period of motion $= 2 \times 365$

$\frac{2\pi}{n} = 730$

$n = \frac{\pi}{365}$

now redraw the graph with everything centred



equation of motion is $x = -1.5 \cos \frac{\pi}{365} t$

($t=0$ $x=-1.5$ at 8·30am -2.4m (ie cos curve)

need to find when is deck level with wharf ie when is deck 2.4n

ie on graph this is at 0·9m

ie $0.9 = -1.5 \cos \frac{\pi}{365} t$

$$\frac{-0.9}{1.5} = \cos \frac{\pi}{365} t$$

$$t = \frac{365}{\pi} \left[2n\pi \pm \cos^{-1}\left(-\frac{9}{15}\right) \right]$$ general solution

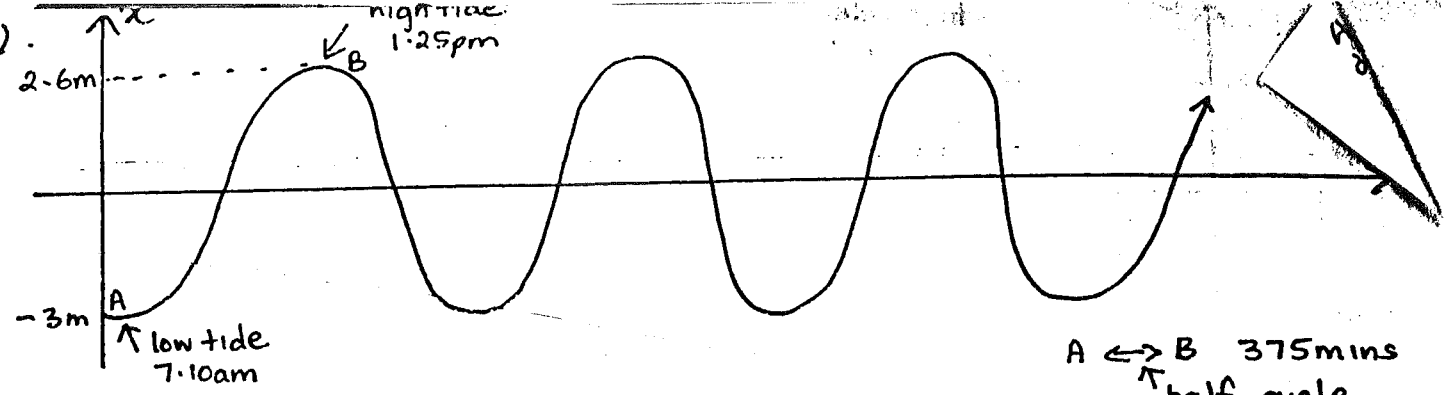
now first occurrence is when $n=0$

ie $t = \frac{365}{\pi} \left[\cos^{-1}\left(-\frac{9}{15}\right) \right]$

$= 257.26$ mins

∴ time deck level with wharf is 8·30am + 257.26mins = 12.47p

(3)



amplitude of motion $2a = 2\cdot6 + 3$

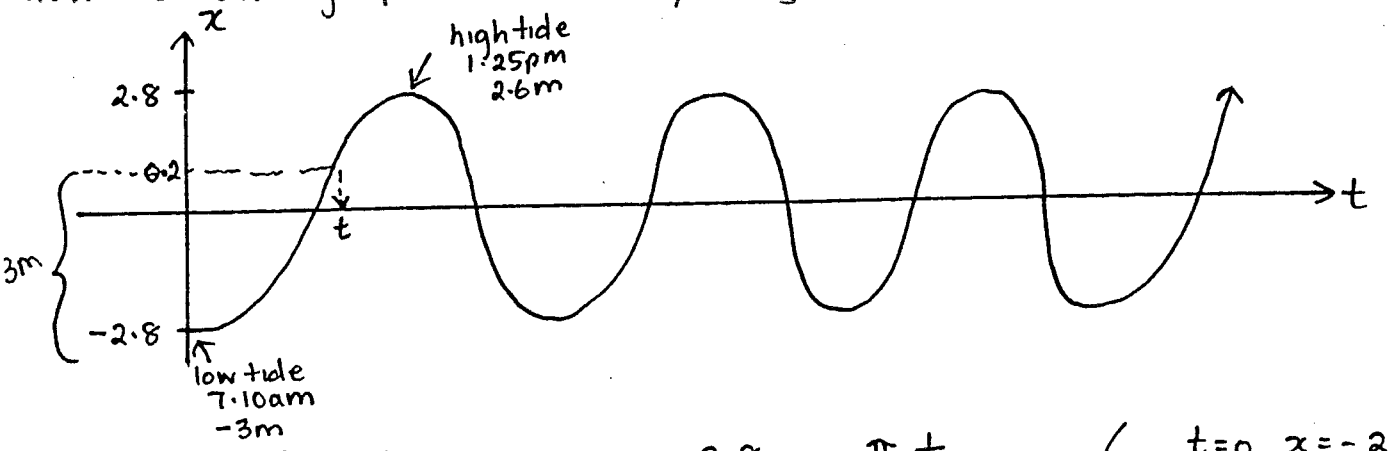
$a = 2\cdot8$

period of motion $= 2 \times 375$

$\frac{2\pi}{n} = 750$

$n = \frac{\pi}{375}$

now redraw graph with everything centred



equation of motion $x = -2\cdot8 \cos \frac{\pi}{375} t$ ($t=0$ $x=-2\cdot8$ at 7·10am $-3m$ ie cos curve)

need to find when is deck level with wharf ie when is deck 3m

ie on graph this is at $x = 0\cdot2m$

ie $0\cdot2 = -2\cdot8 \cos \frac{\pi}{375} t$

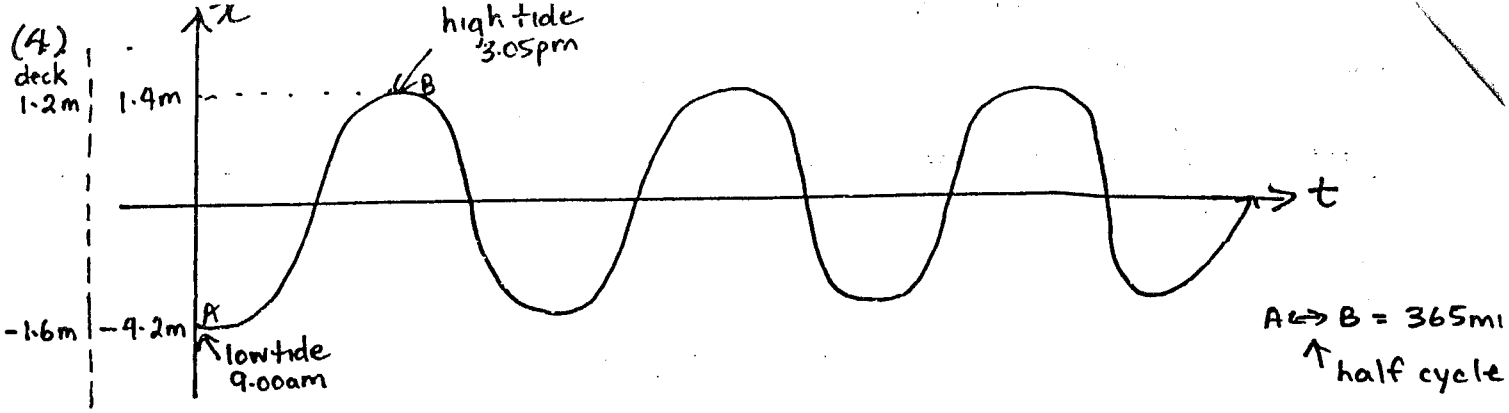
$\frac{\pi}{375} t = \cos^{-1}\left(\frac{-0\cdot2}{2\cdot8}\right)$

$t = \frac{375}{\pi}\left[2n\pi \pm \cos^{-1}\left(-\frac{1}{14}\right)\right]$ general solution

at $n=0$ $t = 196\cdot03$ mins

∴ time deck level with wharf is 7·10am + 196·03mins = 10·26 am

(4)



amplitude of motion (deck) $2a = 1.2 + 1.6$

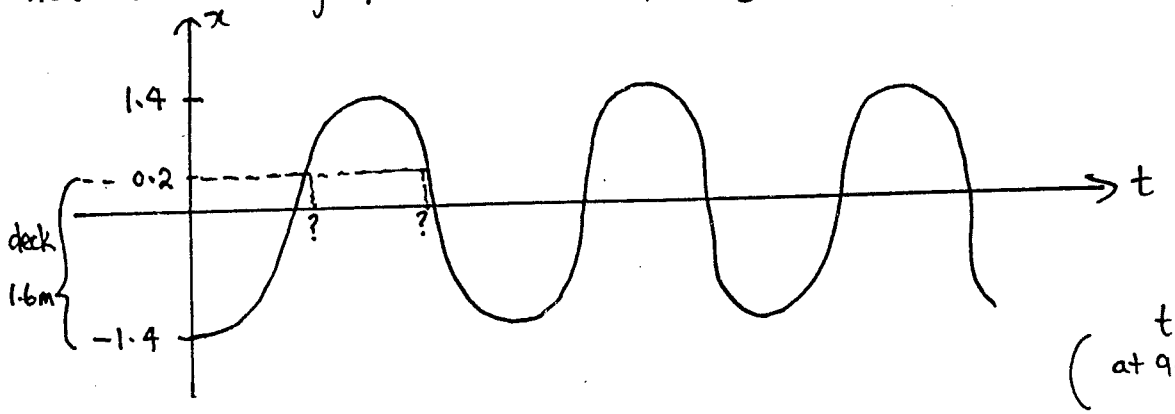
$a = 1.4$

period of motion $= 2 \times 365$

$\frac{2\pi}{n} = 730$

$n = \frac{\pi}{365}$

now redraw graph with everything centred



$t = 0$ at 9.00am $x = -1.4$, -1.6m

(ie cos curve)

equation of motion $x = -1.4 \cos \frac{\pi}{365} t$

need to find when is deck level with wharf ie when is deck 1.6m

ie on graph this is at $x = 0.2$m

ie $0.2 = -1.4 \cos \frac{\pi}{365} t$

$\frac{\pi}{365} t = \cos^{-1}\left(-\frac{1}{7}\right)$

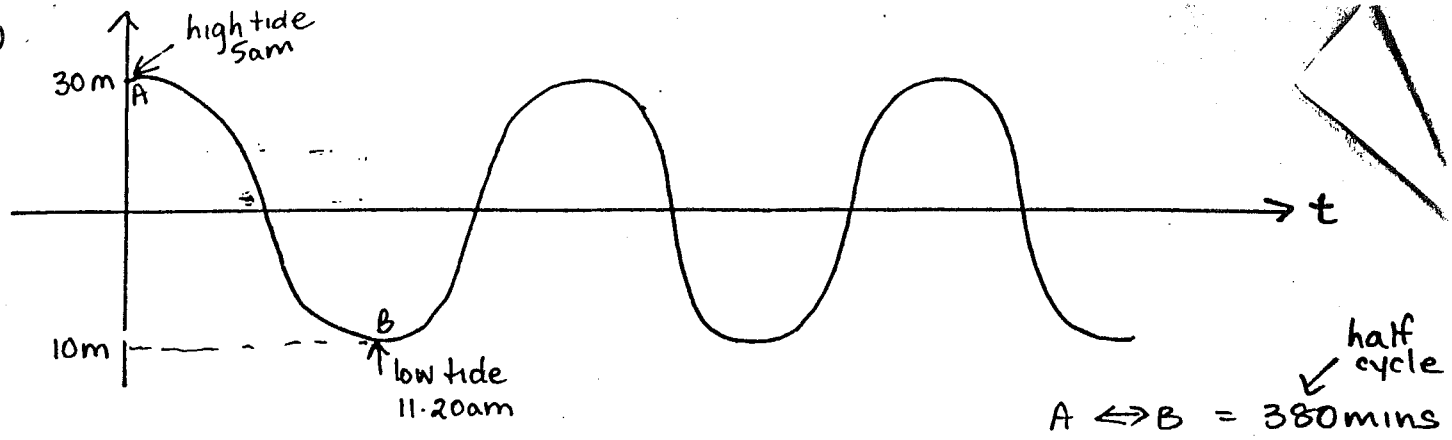
$t = \frac{365}{\pi}\left[2n\pi \pm \cos^{-1}\left(-\frac{1}{7}\right)\right]$ general solution

at $n = 0$ $t = 199.15$ mins at $n = 1$ $t = 531$ mins

ie time deck level with wharf is

9am + 199.15 mins = 12.19pm , 9am + 531 mins = 5.51pm

(5)



A ⟷ B = 380 mins

amplitude of motion $2a = 20$

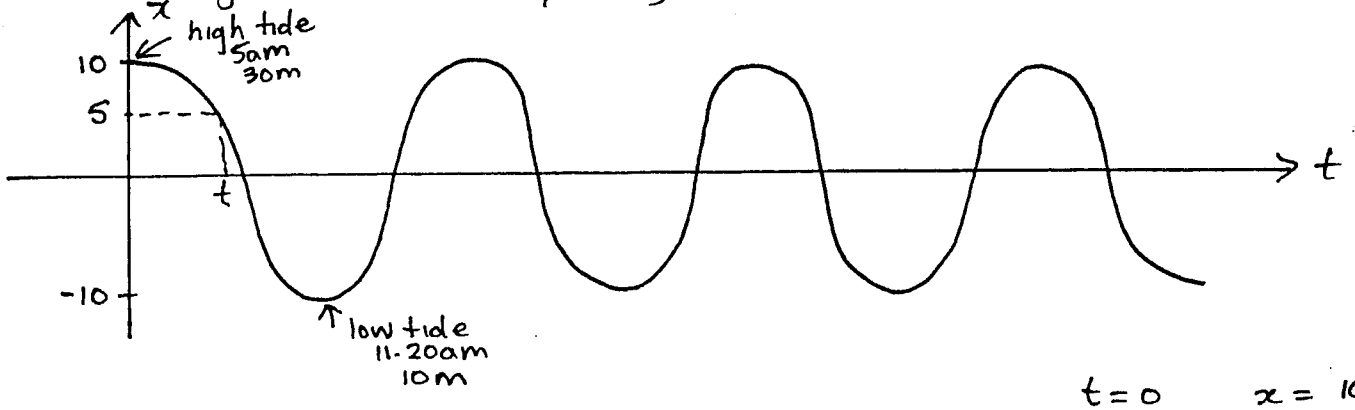
$a = 10$

period of motion $= 2 \times 380$

$\frac{2\pi}{n} = 760$

$n = \frac{\pi}{380}$

now redraw graph with everything centred



$t = 0$ at 5am $\quad x = 10$ 30m

ie cos curve

equation of motion $x = 10 \cos \frac{\pi}{380} t$

now ship drawing 25 metres $\Rightarrow \quad x = 5$

ie $\quad 5 = 10 \cos \frac{\pi}{380} t$

$\frac{\pi}{380} t = \cos^{-1}\left(\frac{1}{2}\right)$

$t = \frac{380}{\pi}\left[2n\pi \pm \cos^{-1}\left(\frac{1}{2}\right)\right]$

when $n=0 \quad t = 126 \cdot 6$ mins

∴ time ship can enter harbour is 5 am + 126·6 mins = 7·07ar