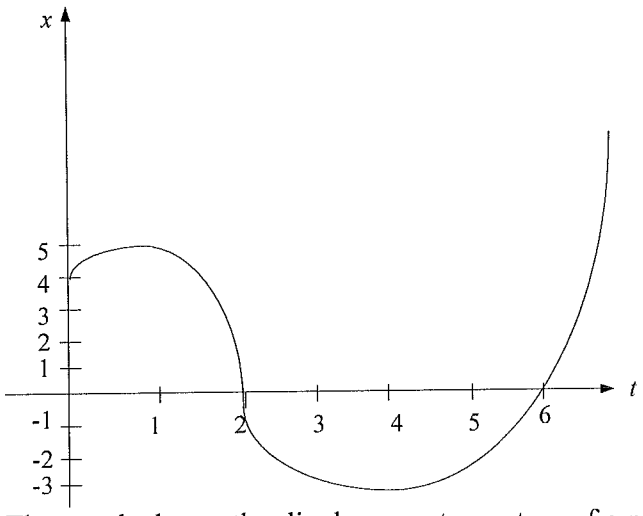
VELOCITY, ACCELERATION AS A FN OF X, SHM & PROJECTILE MOTION

1)



The graph shows the displacement x metres of a particle at time t seconds.

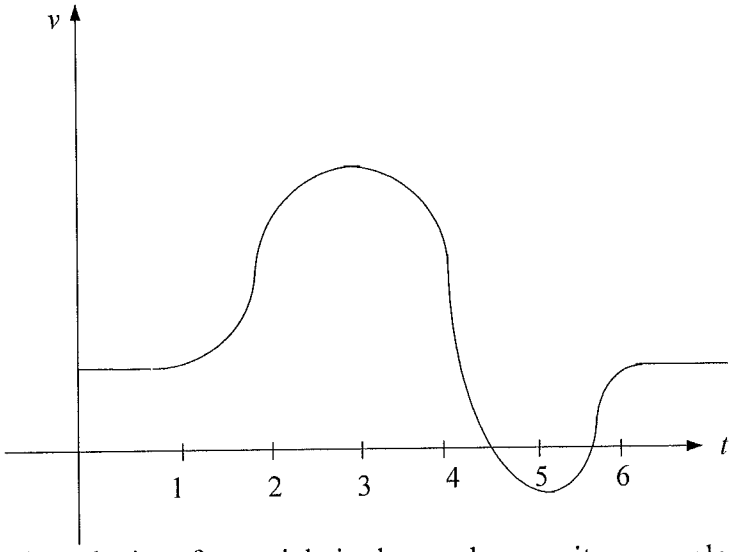
(a) When is the particle at rest?

(b) When is the particle at the origin?

(c) How far does the particle travel in the first 6 seconds?

(d) Draw a rough sketch showing the velocity of the particle.

2)



The velocity of a particle is shown above as it moves along a straight line .

(a) At what times is the acceleration of the particle zero?

(b) Between what times is the acceleration at its greatest?

(c) At what times is the particle at rest?

3) A car starts and drives off, increasing its speed at a constant rate until it reaches 30 m/s after 10 seconds. It travels at this speed for 5 seconds, then it slows down at a constant speed for 5 seconds until it is travelling at 20 m/s. It travels at this speed for 15 seconds.

(a) Sketch the speed v as a function of time t.
(b) Graph the distance travelled by the car over time t on a separate diagram.

4) The displacement of a particle in cm is given by $x = 2t^3 - 21t^2 + 60t$. at time t seconds. Find
(a) the displacement after 3 seconds
(b) the initial velocity
(c) the acceleration after 5 seconds
(d) the times when the particle is at rest
(e) the total displacement over the first 3 seconds.

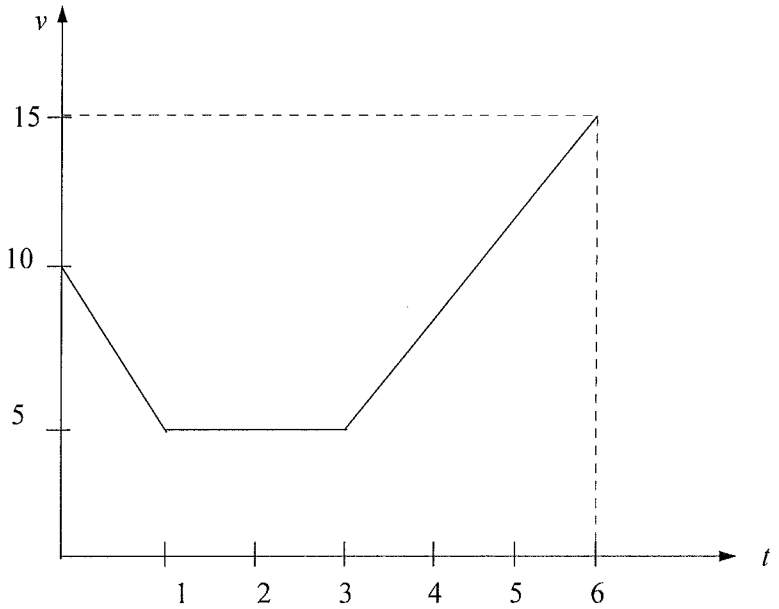
5) The height of a ball in metres over time t seconds is given by $h = 1 + 15t - 5t^2$.
(a) Find the maximum height reached.
(b) Show that the acceleration is constant.
(c) Find the time, to one decimal place, that the ball reaches the ground.

6) The displacement of a spring is given by $s = \cos 2t$ cm at time t seconds. Find
(a) the initial velocity and acceleration
(b) the times when the spring is at rest
(c) the displacement of the spring at these times.
(d) Show that the acceleration of the spring is given by $\ddot{s} = -4s$

7) A particle moves in a straight line so that its displacement is given by $x = 2e^{3t}$ cm at time t seconds. Find
(a) the exact value of the velocity after 9 seconds
(b) the exact time when the acceleration is 36 cms^{-2} .

8) A particle moves along a straight line and its displacement from a fixed point O is given by $x = 5 - 3t + 7\ln(t + 1)$ metres, where t is time measured in seconds.
(a) Find the initial position of the particle.
(b) Find the velocity of the particle after 3 seconds.
(c) Find the acceleration of the particle at time t seconds.

9)



The graph above shows the velocity of a particle in cm/second. Find the total displacement of the particle in the first 6 seconds.

10) The velocity of a particle is $v = 3t^2 - 12t$ ms^{-1} . If the initial displacement is 3 m, find the displacement of the particle after 5 seconds.

11) The acceleration of a particle is given by $a = 18e^{3t}$ cms^{-2}. If the particle is at rest at the origin after 1 second, find the exact displacement of the particle after 3 seconds.

12) Two particles M and N start moving along the x-axis at time $t = 0$. Particle M is initially at $x = 4$ and its velocity at time t is given by $v = 4t - 6$. Particle N has its displacement given by $x = 2t - 2$.
(a) Find the equation of displacement for particle M.
(b) When do the two particles meet?

13) The velocity of a particle after t seconds is $v = 3\cos 3t$ ms^{-1}. The particle is initially 2 m to the left of the origin. Find

(a) the acceleration of the particle after $\frac{\pi}{6}$ seconds

(b) the displacement of the particle after $\frac{\pi}{6}$ seconds.

14) The velocity of a particle is given by $v = \frac{1}{6e^{3x}}$ ms^{-1} where x is the displacement of the particle. If the particle is initially at the origin, find the equation of the displacement of the particle in terms of t.

15) The acceleration of a particle is given by $a = 12x - 6$ ms^{-2}. The particle has an initial velocity of 3 ms^{-1} and is 1 m to the right of the origin. Find the equation for the velocity of the particle.

16) The acceleration of a particle is given by $\ddot{x} = 6x$ ms^{-2}. The particle is travelling at 2 ms^{-1} when it is at the origin. Find the velocity of the particle when it is 5 m to the right of the origin.

17) A pendulum moves so that its displacement in cm over time t seconds is given by $x = \cos 2t$.

(a) Show that its acceleration is $\ddot{x} = -4x$.
(b) Find the times at which the pendulum is at the origin.
(c) Find the velocity when the pendulum is at the origin.
(d) Find its maximum acceleration.

18) A spring moves in simple harmonic motion with acceleration $\frac{d^2x}{dt^2} = -9x$ mms^{-2}.

(a) Show that $x = 5\cos 3t$ is a formula for the displacement of the spring.
(b) Find the amplitude and period of the motion.
(c) What is the maximum speed of the spring?
(d) What is its acceleration when the spring is at the origin?

19) The velocity of a particle moving in SHM is given by $v = 5x - x^2$ cms^{-1}.
(a) Find the two points between which the particle is oscillating.
(b) Find the centre of the motion.

(c) Find the maximum speed of the particle.
(d) Find its acceleration in terms of its displacement x.

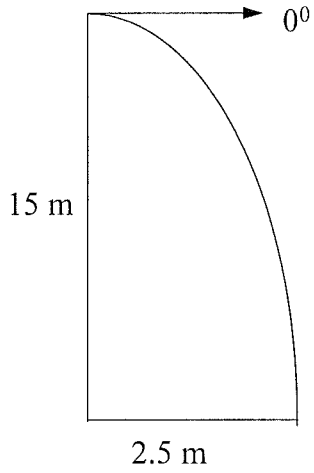
20) A particle is moving in SHM with amplitude 2 cm and period 4 seconds. Find its exact velocity and acceleration when its displacement is 1 cm to the right of the centre of motion.

21) The displacement of a particle is given by $s = 4 \sin 2t + 3 \cos 2t$ cm.
(a) Show that the particle is moving in simple harmonic motion.
(b) Find the amplitude of the particle.
(c) Find its maximum speed.

22) A ball is thrown at 8 ms^{-1} at an angle of 50^0.
(a) Find the ball's maximum height reached.
(b) Find the time taken for the ball to land (to one decimal place).
(c) How far away will the ball land from where it was thrown?
Use 10 ms^{-2} as the acceleration due to gravity and neglect air resistance.

23) Anne fires an arrow at a speed of 25 ms^{-1} and aims at the centre of a target 1.2 m high and 40 metres away. At what angle should Anne fire the arrow so that it hits the centre of the target?
Use 10 ms^{-2} as the acceleration due to gravity and neglect air resistance.

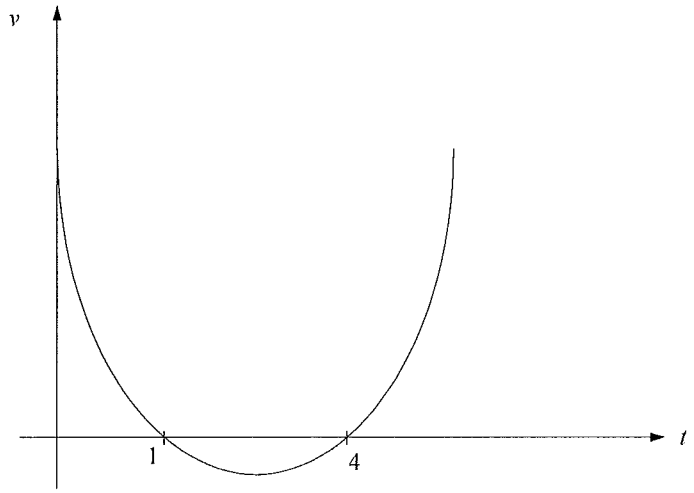
24) Lee drops a book out of a window 15 metres high. He aims at a point on the ground 2.5 metres out from the foot of the building.



What would the initial velocity of the book have to be to hit the ground at that point? Use 9.8 ms^{-2} as the acceleration due to gravity and neglect air resistance.

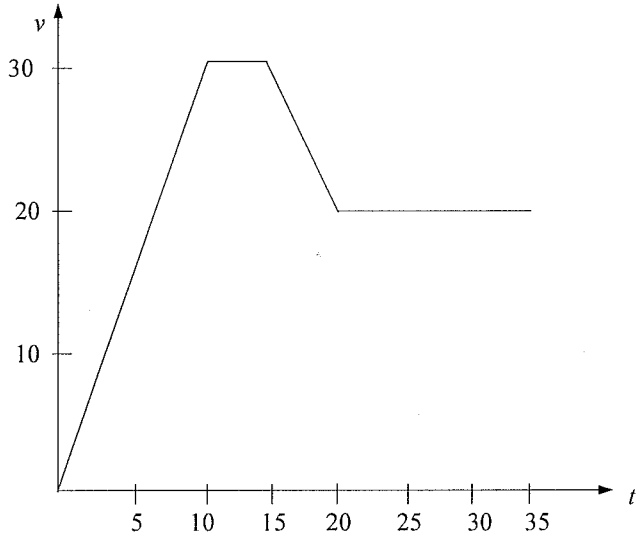
ANSWERS:

1) (a) $t = 1, 4$ s (b) $t = 2, 6$ s (c) 12 metres
(d)

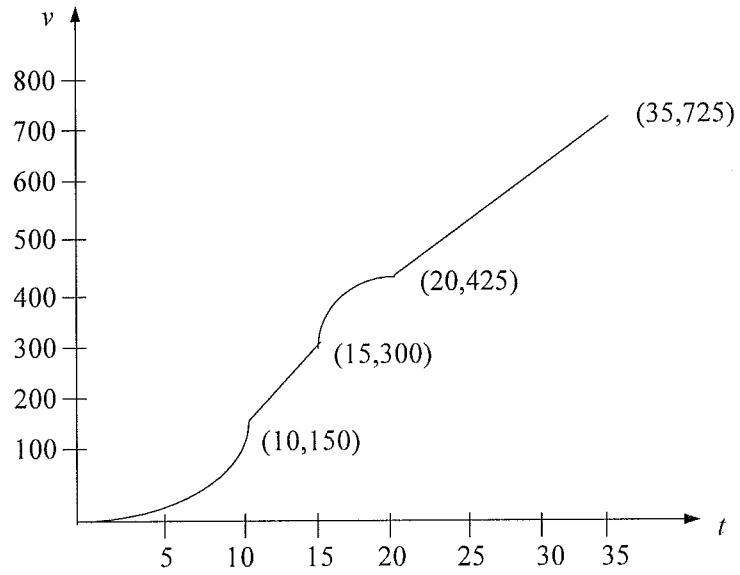


2) (a) $t = 3, 5$ (b) $3 < t < 5$ (c) $0 \leq t \leq 1, t \geq 6$

3) (a)



(b)



4) (a) 45 cm (b) 60 cms^{-1} (c) 18 cms^{-2} (d) t = 2, 5 s (e) 59 cm

5) (a) 12.25 m (b) a = -10 ms^{-2} (c) After 3.1 s

6) (a) $v = 0$ cms^{-1} , a = -4 cms^{-2} (b) $t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$s (c) ±1 cm

(d) $s = \cos 2t$

$\dot{s} = -2\sin 2t$

$\ddot{s} = -4\cos 2t$

$= -4s$

7) (a) $6e^{27}$ cms^{-1} (b) $\frac{\log_e 2}{3}$ s

8) (a) 5 m (b) -1.25 ms^{-1} (c) $-\frac{7}{(t+1)^2}$ ms^{-2}

9) 47.5 cm

10) -22 m

11) $2e^3(e^6 - 7)$ cm

12) (a) $x = 2t^2 - 6t + 4$ (b) $t = 1, 3$ s

13) (a) -9 ms^{-2} (b) -1 m

14) $x = \frac{\ln\left(\frac{t}{2}+1\right)}{3}$

15) $v = \sqrt{12x^2 - 12x + 9}$

16) 12.4 ms^{-1}

17) (a) $x = \cos 2t$

$\dot{x} = -2\sin 2t$

$\ddot{x} = -4\cos 2t$

$= -4x$

(b) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \ldots$ seconds (c) ± 2 cms^{-1} (d) 4 cms^{-2}

18) (a) $x = 5\cos 3t$

$\frac{dx}{dt} = -15\sin 3t$

$\frac{d^2x}{dt^2} = -45\cos 3t$

$= -9(5\cos 3t)$

$= -9x$

(b) Amplitude 5, period $\frac{2\pi}{3}$ (c) 15 mms^{-1} (d) 0 mms^{-2}

19) (a) 0, 5 cm (b) 2.5 cm (c) 6.25 cms^{-1} (d) $a = 25x - 15x^2 + 2x^3$

20) $-\frac{\sqrt{3}\pi}{2} cms^{-1}; -\frac{\pi^2}{4} cms^{-2}$

21) (a) $s = 4\sin 2t + 3\cos 2t$

$\frac{ds}{dt} = 8\cos 2t - 6\sin 2t$

$\frac{d^2s}{dt^2} = -16\sin 2t - 12\cos 2t$

$= -4(4\sin 2t + 3\cos 2t)$

$= -4s$

So the particle is in SHM

(b) 5 (c) 10 cms^{-1}

22) (a) 1.9 m (b) 1.2 s (c) 6.3 m

23) $21^0 53', 69^0 50'$

24) 1.43 ms^{-1}