Volume / Motion Name: _____ Total: /10

A particle is released from rest at an origin on an x axis. Its acceleration is determined by its position so that, at position x, it is 2 cos x.

1. In which direction will it first move?

2. Show that its velocity is given by $v^2 = 4 \sin x$.

3. Where will the particle next come to rest?

4. Describe the subsequent motion.

5. What is the particle's greatest speed?

6. If the particle takes time T to move from $x = \pi/6$ to $x = 5\pi/6$, show that

$$2T = \int_{\pi/6}^{5\pi/5} \sqrt{\operatorname{cosec} x}\ dx.$$

7. Use Simpson's rule with three funtion values to approximate T. (2 decimal places)

Volume / Motion Name: Angelina Total: /10

A particle is released from rest at an origin on an x axis. Its acceleration is determined by its position so that, at position x, it is 2 cos x.

$t=0, x=0, v=0 \quad \ddot{x} = 2\cos x$

1. In which direction will it first move?

Right of axis ✓

2. Show that its velocity is given by $v^2 = 4 \sin x$.

$\ddot{x} = \frac{d}{dx}(\frac{1}{2}v^2) \quad \therefore 2\cos x = \frac{d}{dx}(\frac{1}{2}v^2)$ ✓

$2\sin x = \frac{1}{2}v^2$

$\therefore v^2 = 4\sin x$ ✓

3. Where will the particle next come to rest?

$v = 0, \quad 4\sin x = 0 \quad \therefore$ when $x = \pi$ ✓

4. Describe the subsequent motion.

velocity will increase ✓, and it will decelerate ✓

5. What is the particle's greatest speed?

max |v| when $2\cos x = 0 \quad x = \frac{\pi}{2}, \frac{3\pi}{2}$ ✓ $\quad \frac{d^3x}{dt^3} = -2\sin x$ ✓

when $x = \frac{\pi}{2}, \frac{d^3x}{dt^3} < 0$ & $\frac{d^3x}{dt^3} > 0$ when $x = \frac{3\pi}{2}$

6. If the particle takes time T to move from $x = \pi/6$ to $x = 5\pi/6$, show that $\therefore$ max |v| = 2m/s ✓

$$2T = \int_{\pi/6}^{5\pi/6} \sqrt{\operatorname{cosec} x}\ dx.$$

$\frac{dx}{dt} = 2\sqrt{\sin x}$ ✓

$\frac{dt}{dx} = \frac{1}{2\sqrt{\sin x}}$

$T = \frac{d}{|v|} \quad \therefore 2t = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sqrt{\operatorname{cosec} x}\ dx$ ✓

7. Use Simpson's rule with three funtion values to approximate T. (2 decimal places)

x	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$
f(x)	$\sqrt{2}$	1	$\sqrt{2}$

$\therefore \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sqrt{\operatorname{cosec} x}\ dx \doteq \frac{\frac{2\pi}{3}}{6}(2\sqrt{2}+4)$ ✓

$\therefore 2T \doteq \frac{2\pi}{9}(\sqrt{2}+2)$

$T \doteq \frac{\pi}{9}(\sqrt{2}+2)$ ✓

$\doteq 1.19$ (to 2dp)