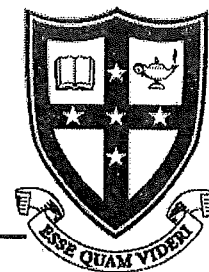


*+ Solus***Question 1 (12 Marks)**

Marked by SKB

Having been despatched to the boundary on the previous two balls, Angas, the crafty medium pace bowling cricketer, then bowls a slower ball at 15 m/s to the elegant right-hander Prinya. Prinya mishits this ball from a point O on the pitch at a velocity of 25m/s inclined at an angle  $\theta$  to the positive direction of the pitch ( $x$  – axis). The ball just misses a bird flying at an altitude of 10m which is 5m horizontally from O. If we assume that air resistance is neglected and that the acceleration due to gravity,  $g = 10 \text{ ms}^{-2}$ :

- a) Prove that the parametric equations describing the ball's motion are:  $x = 25t \cos \theta$  and  $y = 25t \sin \theta - 5t^2$  2
- b) Prove that  $\theta$  satisfies the equation  $\tan^2 \theta - 25 \tan \theta + 51 = 0$  2
- c) Find the value(s) of  $\theta$  to the nearest minute. 2
- d) By using your answer(s) to (c) find the shortest time for the ball to reach the pitch in seconds to 2 decimal places. 2
- e) By using your answer(s) to (c) find the maximum height reached by the ball to the nearest m. 2
- f) Find the possible directions of motion when the ball hits the ground to the nearest minute. 2

**Question 2 (12 Marks)**

Start a new page

Marked by HRK

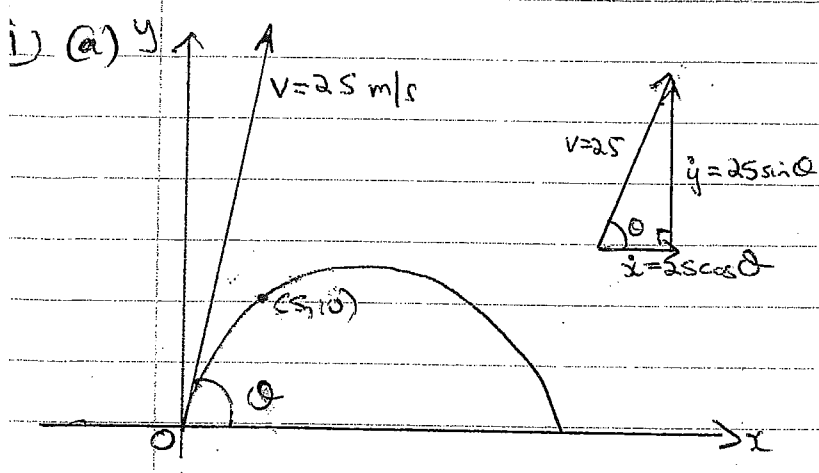
- a) Show that  $\left(x^2 - \frac{1}{x}\right)^{11}$  does not contain a term independent of  $x$ . 3
- b) Find the greatest coefficient in the expansion of  $(2 + 3x)^{10}$  3
- c) Use the binomial expansion of  $(1+x)^{2n}$  to prove that  $\sum_{k=1}^{2n} \binom{2n}{k} = 4^n - 1$  3
- d) An athlete finds that over a long period he wins  $\frac{3}{4}$  of his races. If he intends running in 5 races over the coming weeks, what is the probability that he wins the majority of them? 3

**Question 3 (12 Marks)**

Start a new page

Marked by JSH

- a) How many arrangements of the word GOBBLEDEGOOK are possible? 2
- b) Consider a pack of 40 cards consisting of the colours red, blue, yellow and green, each with cards numbered 1 through to 10. A hand of five cards is dealt from the pack. Find the probabilities that the cards were
- (i) three 4's and two 6's 1
  - (ii) in ascending order 2
- c) Seven people are to be seated at a round table.
- (i) How many arrangements are possible? 1
  - (ii) Two people, Kevin and Jill, refuse to sit next to each other.  
How many seating arrangements are possible? 3
- d) A committee of 3 is to be elected from a club of 8 members.
- (i) How many different committees can be formed? 1
  - (ii) If there are 4 Queenslanders in the club, what is the probability that a randomly selected committee of 3 contains only Queenslanders? 2



Initially  $\ddot{x} = 0$ ,  $\ddot{y} = -10$   
 $\therefore \dot{x} = c_1$ ,  $\dot{y} = -10t + c_2$   
 when  $t = 0$   $\dot{x} = 25 \cos \theta$ ,  $\dot{y} = 25 \sin \theta$   
 $\therefore 25 \cos \theta = c_1$ ,  $25 \sin \theta = c_2$   
 $\therefore \dot{x} = 25 \cos \theta$ ,  $\dot{y} = -10t + 25 \sin \theta$   
 $\therefore x = 25t \cos \theta + c_3$ ,  $y = -5t^2 + 25t \sin \theta + c_4$   
 when  $t = 0$ ,  $x = 0$ ,  $y = 0 \therefore c_3 = 0 = c_4$   
 $\therefore$  parametric equations are:  
 $x = 25t \cos \theta$  — (1)  
 and  $y = 25t \sin \theta - 5t^2$  — (2)

(b) Ball just misses bird at (5, 10)

$\therefore 5 = 25t \cos \theta$  — (1A)  
 $10 = 25t \sin \theta - 5t^2$  — (2A)  
 from (1A)  $t = \frac{1}{5 \cos \theta}$  sub into (2A)  
 $\therefore 10 = 25 \left( \frac{1}{5 \cos \theta} \right) \sin \theta - 5 \left( \frac{1}{5 \cos \theta} \right)^2$

$\therefore 10 = 5 \tan \theta - \frac{\sec^2 \theta}{5}$   
 $\therefore 50 = 25 \tan \theta - (1 + \tan^2 \theta)$   
 $\therefore \tan^2 \theta - 25 \tan \theta + 51 = 0$

(c) Now  $\tan \theta = \frac{25 \pm \sqrt{25^2 - 4 \cdot 1 \cdot 51}}{2}$   
 $= \frac{25 \pm \sqrt{421}}{2}$

$\therefore \theta = 87^\circ 29'$  or  $65^\circ 57'$   
 (to nearest minute)

(d) Ball hits ground when  $y = 0$

$\therefore 0 = 25t \sin \theta - 5t^2$   
 $\therefore 0 = 5t(5 \sin \theta - t)$   
 $\therefore t = 0$  (initial) or  $t = 5 \sin \theta$  (final)

when  $\theta = 87^\circ 29'$   $t = 5.00$  (2dp)  
 $\theta = 65^\circ 57'$   $t = 4.57$  (2dp)

$\therefore$  shortest time for ball is 4.57s (2dp)

(e) At maximum height  $\dot{y} = 0$

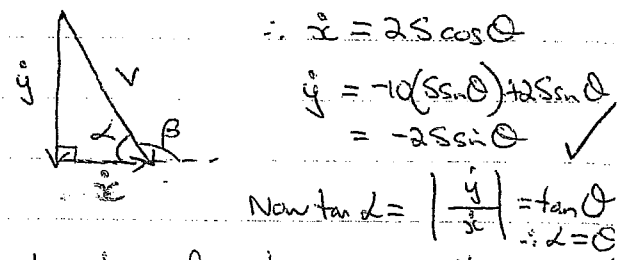
$\therefore t = \frac{25 \sin \theta}{10} = \frac{5 \sin \theta}{2}$

$\therefore y_{\max} = 25 \left( \frac{5 \sin \theta}{2} \right) \sin \theta - 5 \left( \frac{5 \sin \theta}{2} \right)^2$   
 $= \frac{125 \sin^2 \theta}{2} - \frac{125 \sin^2 \theta}{4}$   
 $= \frac{125 \sin^2 \theta}{4}$

Max height will occur for  $\theta = 87^\circ 29'$   
 as  $\sin^2 \theta$  will be greater.

$\therefore y_{\max} = \frac{125 (\sin 87^\circ 29')^2}{4}$   
 $= 31 \text{ m}$  (to nearest m)

(f) Ball hits ground after  $5 \sin \theta$  s from (iv).



$\therefore \beta$ , direction of motion is either  $180^\circ - 87^\circ 29'$  or  $180^\circ - 65^\circ 57'$   
 $\therefore 92^\circ 31'$  or  $114^\circ 3'$  (to nearest minute)

$$2(a) \text{ For } \left(x^2 - \frac{1}{x}\right)^{11}$$

$$\begin{aligned} T_{r+1} &= {}^{11}C_r (x^2)^{11-r} \left(-\frac{1}{x}\right)^r \\ &= {}^{11}C_r x^{22-2r-r} (-1)^r \\ &= {}^{11}C_r x^{22-3r} (-1)^r \end{aligned}$$

For term independent of  $x$ :  $22-3r=0$

$$\therefore r = 7\frac{1}{3}$$

But  $r$  must be a positive integer or zero

$\Rightarrow \left(x^2 - \frac{1}{x}\right)^{11}$  does not contain a term independent of  $x$ .

$$(b) \text{ For } (2+3x)^{10}$$

$$\text{consider } \frac{\text{coeff } T_{r+1}}{\text{coeff } T_r} > 1$$

for greatest coefficient.

$$\therefore \frac{{}^{10}C_r 2^{10-r} (3x)^r}{{}^{10}C_{r-1} 2^{11-r} (3x)^{r-1}} > 1$$

$$\therefore \frac{10! (11-r)! (r-1)! 3}{(10-r)! r! 10! 2} > 1$$

(ignoring  $x$  terms)

$$\therefore \frac{(11-r)3}{(r)2} > 1$$

$$\therefore 33-3r > 2r$$

$$\therefore 33 > 5r \quad \therefore r < 6\frac{3}{5}$$

$$\therefore r = 6$$

$$\therefore \text{greatest coefficient is: } {}^{10}C_6 2^4 3^6 = 2449440$$

$$(c) (1+x)^{2n} = \sum_{k=0}^{2n} \binom{2n}{k} x^k$$

$$\text{Let } x=1 \quad \therefore 2^{2n} = \sum_{k=0}^{2n} \binom{2n}{k} \cdot 1^k$$

$$\therefore 4^n = \binom{2n}{0} + \sum_{k=1}^{2n} \binom{2n}{k} \quad \checkmark$$

$$\therefore 4^n - 1 = \sum_{k=1}^{2n} \binom{2n}{k}, \text{ as } \binom{2n}{0} = 1 \quad \checkmark$$

(d) Consider  $(q+p)^5$

$$\text{where } P(\text{wins}) = p = \frac{3}{4}$$

$$P(\text{loses}) = q = \frac{1}{4}$$

$\therefore P(\text{wins majority of races})$

$$= {}^5C_3 q^2 p^3 + {}^5C_4 q p^4 + {}^5C_5 p^5$$

$$= {}^5C_3 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^3 + {}^5C_4 \left(\frac{1}{4}\right) \left(\frac{3}{4}\right)^4 + \left(\frac{3}{4}\right)^5$$

$$= 10 \left(\frac{27}{1024}\right) + 5 \left(\frac{81}{1024}\right) + \frac{243}{1024}$$

$$= \frac{459}{512} \text{ (or } 0.896484375 \dots)$$

3 (a) COBBLEDEGEEK contains

12 letters including 2 C's, 3 O's, 2 B's

2 E's, 1 L, 1 D, 1 K.

$$\therefore \text{Number of arrangements} = \frac{12!}{2!3!2!2!} \quad \checkmark$$

$$= 9979200$$

$$(b)(i) P(3 \text{ 4's, } 2 \text{ 6's}) = \frac{{}^4C_3 \times {}^4C_2}{{}^{40}C_5}$$

$$= \frac{1}{27417} \quad \checkmark$$

$$(ii) P(\text{in ascending order}) = \frac{[(10-5+1) \times {}^4C_1 \times {}^4C_1 \times {}^4C_1 \times {}^4C_1 \times {}^4C_1]}{{}^{40}C_5}$$

$$= \frac{6 \times 4^5}{{}^{40}C_5}$$

$$= \frac{286}{27417} \quad \checkmark$$

$$\begin{aligned} 3. (c) \quad (i) \quad \text{No. of arrangements} &= 1 \times 6! \\ &= 720 \end{aligned}$$

(ii) Consider Kevin + Jill to sit together. This is achieved in 2! ways. This leaves  $(7-2)+1 = 6$  people units.

$$\therefore \text{No of arrangements Kevin + Jill together} = 1 \times 5! \times 2!$$

$$\begin{aligned} \therefore \text{No of ways Kevin + Jill not together} &= 6! - 1 \times 5! \times 2! \\ &= 480 \end{aligned}$$

$$(d) \quad (i) \quad \text{No. of committees} = {}^8C_3 = 56$$

$$\begin{aligned} (ii) \quad P(\text{only 2'lders}) &= \frac{{}^4C_3}{{}^8C_3} \\ &= \frac{4}{56} \\ &= \frac{1}{14} \end{aligned}$$