1. Which of the following functions are polynomials?

   (a) $5\sqrt{x} + 3x^{-2}$

   (b) $x^3 + 5x^2 - 7x$

   (c) 9

   (d) $5x^{\frac{2}{3}} + 3x - 5$

   (e) $\dfrac{1}{x^3} + x$

   (f) $3^x - x$.

2. Consider the following polynomial $P(x) = (2x^5 + x + 1)(5 - 2x)$:

   (a) What is the leading term?

   (b) What is the coefficient of x^2 ?

   (c) What is the constant term ?

   (d) What is the degree ?

   (e) Find the value of $P(-2)$.

   (f) Is the polynomial monic and why?

3. If $P(x) = 3x + 7$ and $Q(x) = x^3 - 4x + 2$: Find

   (a) $P(x) + Q(x)$

   (b) $P(x) - Q(x)$

   (c) $Q(x) - P(x)$

   (d) $Q(x)P(x)$

4. Find the cubic polynomial that has zeroes 0,1 and 2, and in which the coefficient x^3 is 2.

5. Divide $4x^3 - 4x^2 + 7x + 14$ by $2x + 1$ and write out in terms of the dividend equalling the divisor times the quotient plus the remainder.

   $$P(x) = D(x)Q(x) + R(x)$$

6. Find the remainder when $P(x) = 4x^3 - 7x + 2$ is divided by $x + 2$, using

   (a) long division

   (b) the remainder theorem

7. Find the value of k if $x - 1$ is a factor of $x^3 - 3x^2 + kx - 2$.

8. Solve the equations

   (a) $x^3 + 2x^2 - 9x - 18 = 0$

   (b) $2x^4 + 11x^3 + 19x^2 + 8x - 4 = 0$

9. Sketch the following polynomials by showing any x-intercepts and y-intercept:

   (a) $y = x^2(x-3)(x+3)$

   (b) $y = (x+2)^2(x-1)^3$

   (c) $y = x^3(x+3)(2-x)$

   (d) $y = -x(x-3)^2(x-6)^3$

10. Find the value of a if $3x^4 + ax^2 - 2$ is divisible by $x + 1$.

11. The polynomial $P(x) = x^4 - 2x^3 + ax + b$ has remainder 3 after division with $x - 1$ and has remainder -5 after division by $x + 1$. Find a and b.

POLYNOMIALS

1 b) c) ✓✓

2. $P(x) = (2x^5 + x + 1)(5 - 2x)$

$= 10x^5 - 4x^6 \boxed{+5x} - 2x^2$
$\qquad +5 \boxed{-2x}$

$= -4x^6 + 10x^5 - 2x^2 + 3x + 5$ ✓

a) leading term $= -4x^6$ ✓

b) coefficient of $x^2 = -2$ ✓

c) constant term $= 5$ ✓

d) degree $= 6$ ✓

e) $P(-2) = -4(-2)^6 + 10(-2)^5 - 2(-2)^2$
$\qquad\qquad + 3(-2) + 5$

$= -256 + (-320) - 8 - 6 + 5$

$= -585$ ✓

f) non-monic as ✓
   coefficient of leading term $\neq 1$

3. $P(x) = 3x + 7 \qquad Q(x) = x^3 - 4x + 2$

a) $P(x) + Q(x)$

$= 3x + 7 + x^3 - 4x + 2$

$= x^3 - x + 9$ ✓

b) $P(x) - Q(x)$

$= 3x + 7 - (x^3 - 4x + 2)$

$= 3x + 7 - x^3 + 4x - 2$

$= -x^3 + 7x + 5$ ✓

c) $Q(x) - P(x)$

$= x^3 - 4x + 2 - (3x + 7)$

$= x^3 - 4x + 2 - 3x - 7$

$= x^3 - 7x - 5$ ✓

d) $Q(x) P(x)$

$= (x^3 - 4x + 2)(3x + 7)$

$= 3x^4 + 7x^3 - 12x^2 - 28x$
$\qquad\qquad\qquad + 6x + 14$

$= 3x^4 + 7x^3 - 12x^2 - 22x + 14$ ✓

4. zeroes $x = 0, 1, 2$
    coefficient $x^3 = 2$
    $y = 2x(x-1)(x-2)$ ✓

5.
$$
\begin{array}{r}
2x^2 - 3x \overset{+5}{+2} \\
2x+1 \overline{)\; 4x^3 - 4x^2 + 7x + 14} \\
4x^3 + 2x^2 \\ \hline
-6x^2 - 7x \\
-6x^2 - 3x \\ \hline
10x + 14 \\
10x + 5 \\ \hline
29
\end{array}
$$
✓ ✓

$4x^3 - 4x^2 + 7x + 14$
$= (2x+1)(2x^2 - 3x + 5) + 9$ ✓

6.
$$
\begin{array}{r}
4x^2 - 8x + 9 \\
x+2 \overline{)\; 4x^3 \quad - 7x + 2} \\
4x^3 + 8x^2 \\ \hline
-8x^2 - 7x \\
-8x^2 - 16x \\ \hline
9x + 2 \\
9x + 18 \\ \hline
-16
\end{array}
$$
✓ ✓

$P(-2) = 4(-2)^3 - 7(-2) + 2$

$= 4x - 8 + 14 + 2$

$= -32 + 14 + 2$

$= -16$ ✓

7. $(x-1)$ factor of

$P(x) = x^3 - 3x^2 + kx - 2$

$P(1) = 0$

$P(1) = 1^3 - 3 \times 1 + k - 2$

$\quad = 1 - 3 + k - 2$

$\quad = -2 + k - 2$

$\quad = -4 + k$

$-4 + k = 0$

$\quad k = 4$ ✓

8a) Solve

$x^3 + 2x^2 - 9x - 18 = 0$

Test factors of -18

$x = 2 \qquad 2^3 + 2 \cdot 2^2 - 9(2) - 18 =$

$\qquad\qquad 8 + 8 - 18 - 18 = -40$

$x = -2 \quad (-2)^3 + 2 \cdot (-2)^2 - 9(-2) - 18$

$\qquad\qquad = -8 + 8 + 18 - 18$

$\qquad\qquad = 0 \qquad$ factor $x + 2$ ✓

$$\begin{array}{r} x^2 \qquad - 9 \\ (x+2)\overline{\smash)\; x^3 + 2x^2 - 9x - 18} \\ \underline{x^3 + 2x^2} \\ -9x - 18 \\ \underline{-9x - 18} \end{array}$$

$x^3 + 2x^2 - 9x - 18 = 0$

$(x+2)(x^2 - 9) = 0$

$(x+2)(x-3)(x+3) = 0$

$\quad x = -2, \quad x = 3, \quad x = -3$ ✓

8b) $2x^4 + 11x^3 + 19x^2 + 8x - 4 = 0$

test factors of -4

$x = 1$

$2 \cdot 1 + 11 + 19 + 8 - 4 \neq 0$

$x = -1$

$2 \cdot 1^4 + 11(-1)^3 + 19(-1)^2 + 8(-1) - 4$

$= 2 - 11 - 19 - 18 - 4$

$= -9 - 19 - 18 - 4$

$\neq 0$

$x = -2$

$2 \cdot (-2)^4 + 11(-2)^3 + 19(-2)^2 + 8(-2) - 4$

$= 2 \times 16 + 11 \times -8 + 7 \cdot 6 - 16 - 4$

$= 32 - 88 + 76 - 16 - 4$

$= -56 + 76 - 16 - 4$

$= 0$ ✓

factor $x + 2$

$$\begin{array}{r} 2x^3 + 7x^2 + 5x - 2 \\ (x+2)\overline{\smash)\; 2x^4 + 11x^3 + 19x^2 + 8x - 4} \\ \underline{2x^4 + 4x^3} \\ 7x^3 + 19x^2 \\ \underline{7x^3 + 14x^2} \\ 5x^2 + 8x \\ \underline{5x^2 + 10x} \\ -2x - 4 \\ \underline{-2x - 4} \end{array}$$ ✓

Let

$Q(x) = 2x^3 + 7x^2 + 5x - 2$

test factors of -2

$Q(2) = 2 \cdot 2^3 + 7 \cdot 2^2 + 5 \cdot 2 - 2$

$\qquad = 16 + 28 + 10 - 2$

$\qquad \neq 0$

4 $\qquad Q(-2) = 2(-2)^3 + 7(-2)^2 + 5(-2) - 2 = 0$ ✓

$\qquad\qquad -16 + 28 + 10 - 2$

$$\frac{2x^2 + 3x - 1}{}$$

$$x+2 \overline{\smash{\big)}\ 2x^3 + 7x^2 + 5x - 2}$$
$$\underline{2x^3 + 4x^2}$$
$$3x^2 + 5x$$
$$\underline{3x^2 + 6x}$$
$$-x - 2$$
$$\underline{-x - 2} \quad \checkmark$$

$$2x^2 + 3x - 1$$
$$\Delta = b^2 - 4ac \qquad a = 2$$
$$= 3^2 - 4 \cdot 2 \cdot -1 \qquad b = 3$$
$$= 9 + 8 \qquad\qquad c = -1$$
$$= 17$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$
$$= \frac{-3 \pm \sqrt{17}}{4} \qquad \checkmark$$

$$x = -2, \quad -\frac{3 \pm \sqrt{17}}{4}$$

1. a) $y = x^2(x-3)(x+3)$

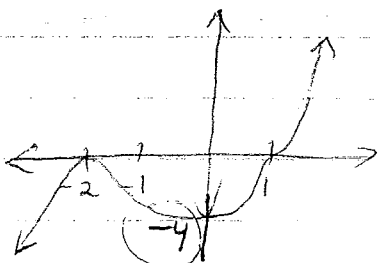$y = 0$ $x = 0, 0, 3, -3$

double root $x = 0$

$\checkmark$
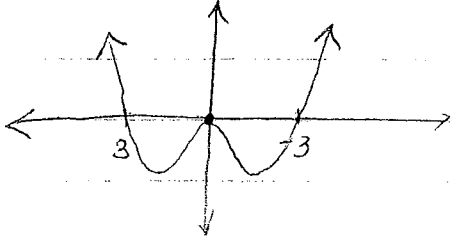
b) $y = (x+2)^2 (x-1)^3$

double root   triple root
$x = -2$ $x = 1$

$\checkmark$

$x = 0$

$x = 0$
$y = 2^2 \times (-1)^3$
$= -4 \quad /6$

c) $y = x^3(x+3)(2-x)$

$y = 0$
  $x = 0, -3, 2$
  triple root at $x = 0$

$\checkmark\checkmark$

$x = 1$
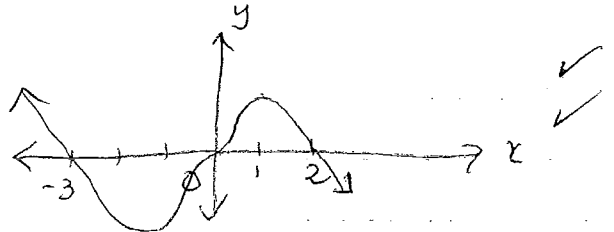
$y = 1^3 (1+3)(2-1)$
$= 1 \times 4 \times 1$

d) $y = -x(x-3)^2(x-6)^3$

$y = 0$ $x = 0, 3, 6$
  double root at $x = 3$
  triple root at $x = 6$

$\checkmark\checkmark$
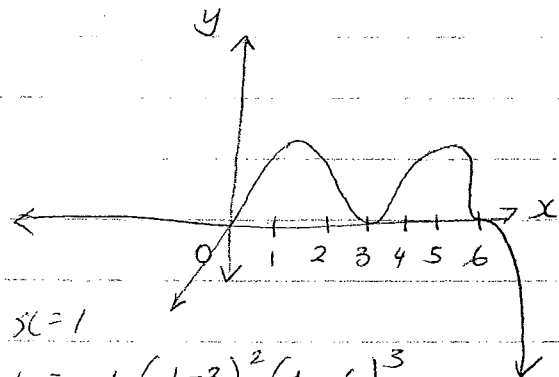
$x = 1$
$y = -1(1-3)^2(1-6)^3$
$= -1 \cdot -2^2 \cdot -5^3$
$= -1 \cdot 4 \cdot -125$
$= 500$
> 0

$/4$

$\underline{P(x) =}$

10. $3x^4 + ax^2 - 2$

    factor $\quad x + 1$

    $\therefore \quad P(-1) = 0$

    $3 \cdot (-1)^4 + a(-1)^2 - 2$

    $3 + a - 2 = 0$

    $a + 1 = 0$

    $a = -1 \qquad \checkmark$

11. $P(1) = 3$

    $P(-1) = -5$

    $P(x) = x^4 - 2x^3 + ax + b$

    $P(1) = 1^4 - 2(1)^3 + a(1) + b$

    $\quad = 1 - 2 + a + b$

    $\quad = -1 + a + b$

    $-1 + a + b = 3$

    $\underline{\quad a + b = 4 \qquad \text{①}}$

    $P(-1) = (-1)^4 - 2(-1)^3 + a(-1) + b$

    $\quad = 1 + 2 - a + b$

    $1 + 2 - a + b = -5$

    $3 - a + b = -5$

    $\underline{-a + b = -8 \qquad \text{②}}$

    $\underline{\text{①a)} \qquad a = 4 - b \qquad \text{① + ②}}$

    $\underline{\text{sub } 1a \text{ in ②}}$

    $2b = -4$

    $b = -2$

    $a = 6 \qquad \checkmark$

    $\diagup 3$