

KESSER TORAH COLLEGE

Assessment 1 – Mathematics Advanced and Extension 1

28<sup>th</sup> February 2011



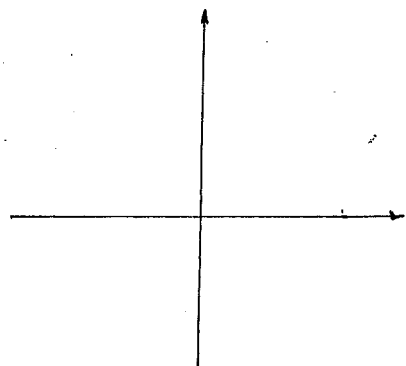
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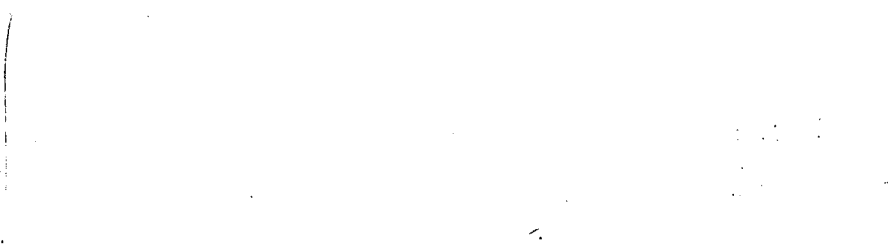
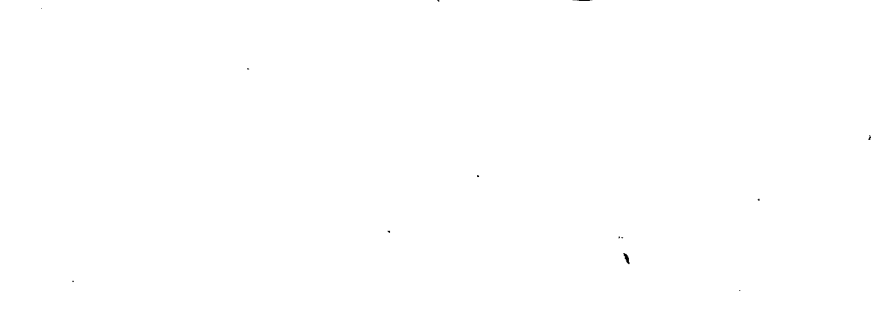


Time Allowed: 60 minutes

Question	Marks
1	15
2	15
3 (Extension 1 only)	9
4	14 (16 for extension 1)

Question 1 (15 Marks)	
Answer in the space provided. Show all working.	
a)	Express the following as integers. (1,1)  i. Calculate $\frac{\sqrt{22.8} \times 5.43}{6.32 + 4.56}$ correct to three significant figures.  ii.
b)	Factorise $2a^2 + 11a + 15$ . (2)
c)	Factorize and simplify: $\frac{t^2 - 9}{3t - 9}$ . (2)
d)	Express 0.36 as a fraction in lowest possible terms. (2)

e)	Factorise $8p^3 + 64$ . (2) 
f)	Solve the equation: $\frac{x}{3} - \frac{2x+1}{4} = 5$ . (2) 
g)	By completing the square determine the radius and centre of the circle $x^2 - 4x + y^2 + 6y + 4 = 0$ and then sketch the it on the axes below, labelling all critical features. (3) 

<b>Question 2 (15 Marks)</b> Answer in the space provided. Show all working.	
a)	Factorise fully $x(x-1)^2 + x^2(x-1)$ . (2) 
b)	Simplify $\frac{2x^2 - 2x - 4}{(x+1)^2(x-2)}$ . (2) 
c)	Let $f(x) = \frac{x}{x^2-1}$ (2) i) What is the natural domain of $f(x)$ ? ii) Is the function odd, even or neither? State your reasons.
d)	Simplify and express with a rational denominator: $\frac{1}{\sqrt{5}} + \sqrt{40} + \frac{4}{\sqrt{80}}$ . (3) 

e)	Graph on the number line the solution set of: $\frac{x-1}{2} - \frac{2x-3}{3} < 1$ . (3)
f)	The volume $V$ of metal in a pipe is given by: $V = L\pi(R+r)(R-r)$ where $L$ is the length of the pipe, $R$ the external radius and $r$ the internal radius. A pipe of length 49 cm and internal radius 1.5 cm is made out of 616 cm <sup>3</sup> of metal. Find the external radius of this pipe. Give your answer to the nearest millimetre. (3)

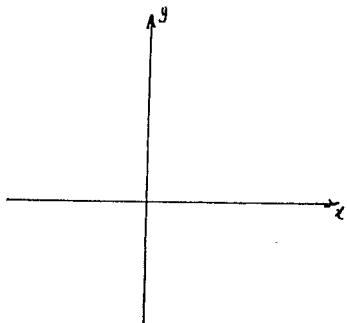
Question 3(Ext) 4 Marks Answer in the space provided. Show all working.	
EXT a)	Find the values of $x$ and $y$ if: $6 + \sqrt{x-y} = x + y + 3\sqrt{2}$ (3)
b)	Factorize $2^{n+1} + 2^n$ , and hence write $\frac{2^{1001} + 2^{1000}}{3}$ as a power of 2. (3)
c)	Solve the inequality $\frac{x^2-4}{x} > 0$ . (3)

Question 4

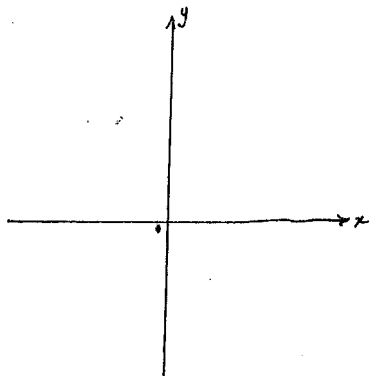
Sketch each of the following curves on separate graphs, clearly showing all significant points (intercepts, vertices, asymptotes) using transformations, shifting or any other convenient method.

State the domain and range of each function. (2 marks each)

i)  $-5x + 2y + 15 = 0$



ii)  $y = -x^2 + 4x + 12$



iii)  $y = \frac{1}{x-2} + 2$

iv)  $y = (x+1)^3$

v)  $y = 4 - \sqrt{x}$

$$\text{vi) } y = \sqrt{4 - x^2}$$

$$\text{vii) } y = 3 + 2^{-x}$$

viii) (Extension 1 only – Mathematics Advanced may attempt for bonus points)

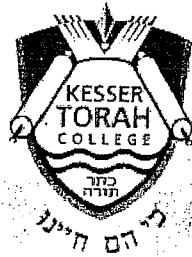
$$y = \frac{1}{x^2}$$

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Assessment 1 – Mathematics Advanced and Extension 1

28<sup>th</sup> February 2011

Name: \_\_\_\_\_



Time Allowed: 60 minutes

Question	Marks
1	15
2	15
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4	14 (16 for extension 1)

Question 1 (15 Marks)

Answer in the space provided. Show all working.

37.5/55 This is a very pleasing result. You are working very well.

a) Express the following as integers. (1,1)

i. Calculate  $\frac{\sqrt{22.8} \times 5.43}{6.32 + 4.56}$  correct to three significant figures.  
 $0.754 \times$

ii.  $\frac{21}{\sqrt{63}} - \frac{3}{\sqrt{7+2}} \cdot \frac{\sqrt{3}+2}{\sqrt{3}-2}$  ✓  
 $\frac{21\sqrt{63}}{63} - \frac{3\sqrt{3}-6}{7-4} = 2 \frac{\sqrt{63}}{3} - \frac{(3\sqrt{3}-6)}{3}$  ✓  
 $\frac{3\sqrt{7}-3\sqrt{3}-6}{3} = \frac{-6}{3} = -2$  ✓

b) Factorise  $2a^2 + 11a + 15$ . (2)  
 $(2a + 5)(a + 3)$  ✓  
 $2a \times 5$   
 $a \times 3$

c) Factorize and simplify:  $\frac{t^2-9}{3t-9}$ . (2)  
 $\frac{(t-3)(t+3)}{3(t-3)} = \frac{t+3}{3}$  ✓

d) Express 0.36 as a fraction in lowest possible terms. (2)  
 $100x - 100x = 36.66 - 3.66$   
 $90x = 33$   
 $x = \frac{33}{90}$   
 $x = \frac{11}{30}$   
 $0.36 = x$   
 $3.6 = 10x$   
 $36.6 = 100x$   
 $100x - x = 36.6 - 0.36$   
 $99x =$   
 $0.36 = \frac{11}{30}$  ✓

e) Factorise  $8p^3 + 64$ . (2) 1/2

$$(2p + 4)(4p^2 + 8p + 16)$$

$$2(p+2)4(p^2 - 2p + 4)$$

$$= 8(p+2)(p^2 - 2p + 4)$$


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f) Solve the equation:  $\frac{x}{3} - \frac{2x+1}{4} = 5$ . (2)

$$\frac{4(x)}{12} - \frac{3(2x+1)}{12} = 5$$

$$\frac{4x - 6x - 3}{12} = 5$$

$$\frac{-2x - 3}{12} = 5$$

$$-2x - 3 = 60$$

$$-2x = 63$$

$$x = \frac{63}{-2}$$


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g) By completing the square determine the radius and centre of the circle  $x^2 - 4x + y^2 + 6y + 4 = 0$  and then sketch it on the axes below, labelling all critical features. (3)

$$x^2 - 4x + 4 + y^2 + 6y + 9 = -4 + 4 + 9$$

$$(x-2)^2 + (y+3)^2 = 9$$

radius = 3

your drawing should be a bit clearer.

2/3

**Question 2 (15 Marks)**  
Answer in the space provided. Show all working.

a) Factorise fully  $x(x-1)^2 + x^2(x-1)$ . (2) let  $(x-1) = A$  & sub in  $x A^2 + x^2 A$

$$[x(x+x^2)(x-1)] \times$$

$$= xA(A+x)$$

$$x(x^2 - 2x + 1) + x^3 - x^2 \text{ sub back}$$

$$x^3 - 2x^2 + x + x^3 - x^2 = x(x-1)(x-1+x)$$

$$= x(x-1)(2x-1)$$


---

b) Simplify  $\frac{2x^2 - 2x - 4}{(x+1)^2(x-2)}$ . (2) 1/2

$$\frac{(2x-4)(x-1)}{(x+1)(x-2)}$$

incomplete

$$= \frac{2(x-2)}{(x+1)(x-2)} \Rightarrow \frac{2}{x+1}$$


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c) Let  $f(x) = \frac{x}{x^2-1}$ . (2)

i) What is the natural domain of  $f(x)$ ?  $x^2 - 1 \neq 0$   
 $x^2 \neq 1$   
 $x \neq \pm 1$

ii) Is the function odd, even or neither? State your reasons.

$$f(-x) = \frac{-x}{(-x)^2 - 1} = \frac{-x}{x^2 - 1} = -f(x)$$

not even (see ①)  
odd (see ②)

---

d) Simplify and express with a rational denominator:  $\frac{1}{\sqrt{5}} + \sqrt{40} + \frac{4}{\sqrt{80}}$ . (3)

$$= \frac{\sqrt{5}}{5} + \frac{\sqrt{40}}{1} + \frac{4\sqrt{80}}{80}$$

$$\frac{16\sqrt{5}}{80} + \frac{80\sqrt{40}}{80} + \frac{4\sqrt{80}}{80}$$

$$= \frac{16\sqrt{5} + 80\sqrt{40} + 4\sqrt{80}}{80}$$

$$\sqrt{40} = \sqrt{5 \cdot 4 \cdot 2} = 2\sqrt{10}$$

$$= \frac{16\sqrt{5} + 160\sqrt{10} + 4\sqrt{5}}{80}$$

$$= \frac{20\sqrt{5} + 160\sqrt{10}}{80}$$

$$= \frac{\sqrt{5} + 2\sqrt{10}}{4}$$

2/3

e)

Graph on the number line the solution set of:  $\frac{x-1}{2} - \frac{2x-3}{3} < 1$ . (3)

$$\frac{3(x-1)}{6} - \frac{2(2x-3)}{6} < 1$$

$$\frac{3x-3}{6} - \frac{4x+6}{6} < 1$$

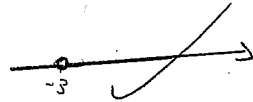
$$\frac{3x-3-4x+6}{6} < 1$$

$$-\frac{x+3}{6} < 1$$

$$-x+3 < 6$$

$$-x < 3$$

$$x > -3$$



f)

The volume  $V$  of metal in a pipe is given by:  $V = L\pi(R+r)(R-r)$  where  $L$  is the length of the pipe,  $R$  the external radius and  $r$  the internal radius. A pipe of length 49 cm and internal radius 1.5 cm is made out of  $616 \text{ cm}^3$  of metal. Find the external radius of this pipe. Give your answer to the nearest millimetre. (3)

$$V = (49 \times \pi) (R + 1.5)(R - 1.5)$$

$$616 \text{ cm}^3 = (49 \times \pi) (R^2 - 2.25)$$

$$4.001609 \dots = (R^2 - 2.25)$$

$$= R^2$$

$$R^2 = 4.001609 + 2.25$$

$$R = \sqrt{4.001609 + 2.25}$$

2/3

Question 3(Ext) 4 Marks

Answer in the space provided. Show all working.

EXT

a) Find the values of  $x$  and  $y$  if:

$$6 + \sqrt{x-y} = x+y+3\sqrt{2} \quad (3)$$

$$6 - 2y = 2$$

$$x + y = 6 \quad (1)$$

$$-2y = -4$$

$$\sqrt{x-y} = \sqrt{2} \quad (2)$$

$$y = 2$$

$$\sqrt{x-y} = 3\sqrt{2} = \sqrt{18} \quad x = 4$$

$$x = 6 - y \quad x - y = 18$$

$$6 - y - y = \sqrt{2}$$

$$6 - 2y = \sqrt{2}$$

1/3

b)

Factorize  $2^{n+1} + 2^n$ , and hence write  $\frac{2^{1001} + 2^{1000}}{3}$  as a power of 2. (3)

$$2^n(2^1 + 1)$$

$$\frac{2^{1000}(2^1 + 1)}{3} \quad \text{incomplete}$$

$$= 2^{1000}$$

2/3

c)

Solve the inequality  $\frac{x^2-4}{x} > 0$ . (3)

$$x \neq 0$$

$$\frac{x^2-4}{x} > 0$$

$$\left[ \begin{array}{l} x(x^2-4) > 0 \\ x^3-4x > 0 \end{array} \right]$$

$$x^4 - 4x^2 > 0$$

$$(x^2-4x)(x^2+4x) > 0$$

$$x(x^2-4) > 0$$

$$x^2 - 4x > 0$$

$$x^2 > 4x$$

$$x(x-2)(x+2) > 0$$

then sketch to see where  $y > 0$

$$\frac{x^2-4}{x} > 0$$

We solve inequations by sketching & reading the solution off the graph.

$$(x^2-4)(x^2) > 0$$

$$(x^2-2)(x^2+2) > 0$$

$$x^2$$

1/3

10/15

4/12



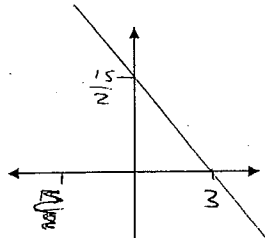
**Question 4**

Sketch each of the following curves on separate graphs, clearly showing all significant points (intercepts, vertices, asymptotes) using transformations, shifting or any other convenient method.

State the domain and range of each function. (2 marks each)

i)  $-5x + 2y + 15 = 0$

Let  $x = 0$   
 $2y + 15 = 0$   
 $2y = -15$   
 $y = -\frac{15}{2}$  ✓



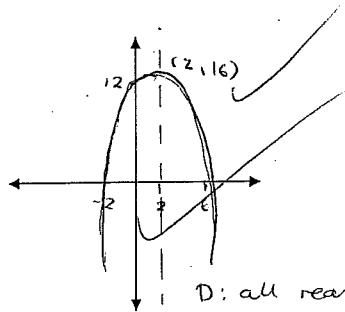
1.5/2

Let  $y = 0$   
 $-5x + 15 = 0$   
 $-5x = -15$   
 $x = 3$

D: all real  $x$   
 R: all real  $y$

ii)  $y = -x^2 + 4x + 12$

Let  $x = 0$   
 $y = 12$



1.5/2

D: all real  $x$   
 R: all real  $y$   $y \leq 16$

Let  $y = 0$   
 $0 = -x^2 + 4x + 12$   
 $x^2 - 4x - 12 = 0$   
 $(x - 6)(x + 2) = 0$   
 $x = 6, -2$

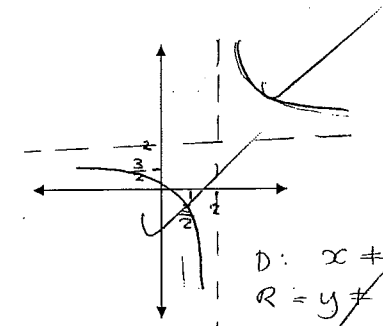
$\frac{-b}{2a} = x$   
 $\frac{-4}{2(-1)} = x$   
 $\frac{-4}{-2} = x$   
 $x = 2$

$y = -(-2)^2 + 4(-2) + 12$   
 $y = 16$

~~$\frac{1}{x-2} = -2$~~   
 $-3 = -2x$   
 $x = \frac{3}{2}$

iii)  $y = \frac{1}{x-2} + 2$

Let  $x = 0$   
 $y = -\frac{1}{2} + 2$   
 $y = \frac{3}{2}$

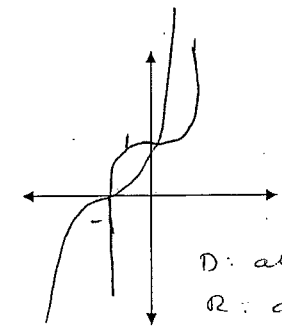


2/2

Let  $y = 0$   
 $\frac{1}{x-2} + 2 = 0$   
 $\frac{1}{x-2} = -2$   
 iv)  $y = (x+1)^3$

D:  $x \neq 2$   
 R:  $y \neq 2$

Let  $x = 0$   
 $y = 1$

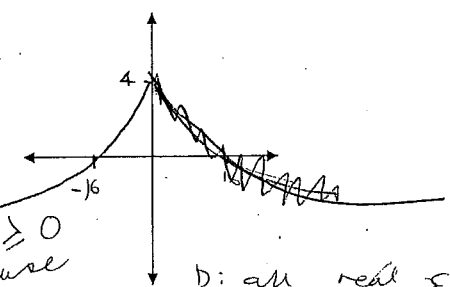


1.5/2

D: all real  $x$   
 R: all real  $y$

v)  $y = 4 - \sqrt{x}$

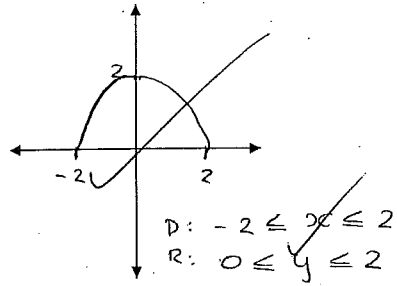
Let  $y = 0$   
 $-4 = -\sqrt{x}$   
 $\sqrt{x} = 4$   
 $x = 16$



$x \geq 0$   
 because it's in a  $\sqrt{\quad}$  sign

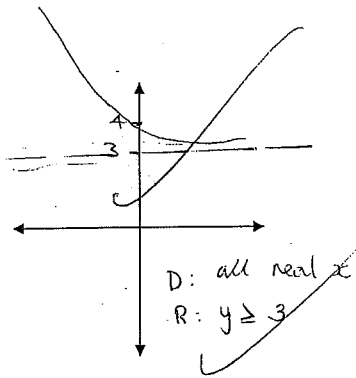
D: all real  $x$   
 R: all real  $y$   
 $x \geq 0$   
 $y \leq 4$

vi)  $y = \sqrt{4-x^2}$



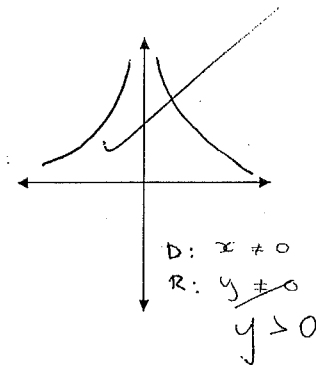
vii)  $y = 3 + 2^{-x}$

at  $x = 0$   
 $y = 4$



viii) (Extension 1 only – Mathematics Advanced may attempt for bonus points)

$y = \frac{1}{x^2}$



1.5/2

Great result!  
 ☺  
 12/16