J.M.J.Ch

MARCELLIN COLLEGE RANDWICK

YEAR 11

ACCELERATED MATHEMATICS
Assessment Task #2

2011

Weighting: 70 % towards Preliminary mark

STUDENT NAME: _ _ _ **MARK:** / 36

Time Allowed: **60 minutes.**

Directions:

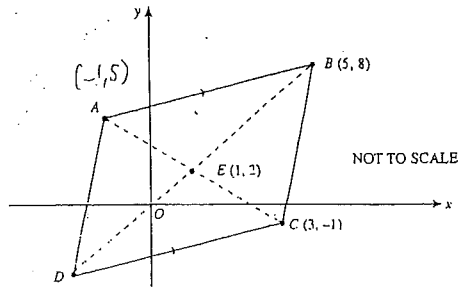
- Answer all questions.
- Begin each question on a new page.
- Show working where necessary.
- Marks may not be awarded for answers only.

OUTCOMES TO BE ASSESSED:

- **P1** – demonstrates confidence in using mathematics to obtain realistic solutions to problems.
- **P2** – provides reasoning to support conclusions which are appropriate to the context.
- **P3** – performs routine arithmetic and algebraic manipulation involving trigonometric identities.
- **P5** – understands the concept of a function and the relationship between a function and its graph.

Question 1 (12 marks)

a.



The diagram shows a parallelogram $ABCD$. The points $E(1,2)$, $B(5,8)$ and $C(3,-1)$ are shown on the number plane.

Copy the diagram on to your answer sheet.

i. E is the midpoint of AC. Show that the coordinates of A are (-1,5). **2**

ii. Find the gradient of the line AB. **1**

iii. Show that the equation of DC is $x - 2y - 5 = 0$. **2**

iv. Calculate the length of AB in the form $k\sqrt{5}$ units. **1**

v. Calculate the perpendicular distance between the lines DC and AB. **2**

vi. Hence, or otherwise, calculate the area of the parallelogram $ABCD$. **1**

b. Find the equation of a line passing through the midpoint of the interval with endpoints $A(2,3)$ and $B(5,7)$ parallel to the line $2x + 3y - 7 = 0$. **3**

Question 2 (12 marks)

a. Consider the quadratic equation $x^2 - kx + (k+3) = 0$.

i. Find the discriminant leaving your answer in simplest form. **1**

ii. For what values of k does the equation have no real roots? **1**

iii. If the product of the roots is equal to three times the sum of the roots, find the value of k. **2**

b. If α and β are the roots of the quadratic equation $2x^2 - x - 5 = 0$, find:

i. $\alpha + \beta$ **1**

ii. $\alpha\beta$ **1**

iii. $(\alpha - 2)(\beta - 2)$ **2**

c. Solve $4\cos^2\vartheta + 2\sin\vartheta = 3$ for $0° \le \vartheta \le 360°$ correct to the nearest minute. **4**

Question 3 (12 marks)

a. Find the centre and radius of the circle whose equation is $x^2 + y^2 + 4x + 8y + 11 = 0$. **3**

b. A parabola has the equation $x^2 - 6x - 6y - 3 = 0$.

Find:

i. the focal length. **2**

ii. the coordinates of the vertex. **1**

iii. the coordinates of the focus. **1**

iv. the equation of the directrix. **1**

c. A point $P(x, y)$ moves so that its distance from the point $A(2,5)$ is twice its distance from the line $x = -1$. Draw a diagram and find the equation of the locus P. **4**

Question 1 (10)

$\frac{x-3}{2} = 0$? $\quad \frac{y-1}{2} = 2$

$x-3 = -2$ $\quad y-1 = 4$ ✓

$x = -1$ $\quad y = 5$

$\therefore (-1, 5) = H$

ii $m = \frac{8-5}{5+1}$

$= \frac{3}{6} = \frac{1}{2}$ ✓

ii $m_{AB} = m_{DC}$ (parallel lines)

$\therefore y = \frac{1}{2}x + b$

$-1 = \frac{1}{2}(3) + b$ ✓

$-1 = 1\frac{1}{2} + b$

$b = -2\frac{1}{2}$

$\therefore y = \frac{1}{2}x - 2\frac{1}{2}$ ✓

$2y = x - 5$

$0 = x - 2y - 5$

iii) $\frac{|5 - 16 - 5|}{\sqrt{1^2 + 2^2}}$ ✓

$= \frac{16}{\sqrt{5}}$ ✓

$= \frac{16\sqrt{5}}{5}$

iv) $d = \sqrt{(5+1)^2 + (8-5)^2}$

$= \sqrt{45}$ ✓

$= 3\sqrt{5}$

Since $AB = 3\sqrt{5}$ and perp. distance $= \frac{16\sqrt{5}}{5}$

$\therefore 3\sqrt{5} \times \frac{16\sqrt{5}}{5}$

$= \frac{48+5}{5}$

$= 48$ sq units^2 ✓

B $m_1 = m_2$

$\therefore m_1 = -\frac{2}{3}$ ✓

$\therefore y = -\frac{2}{3}x + b$

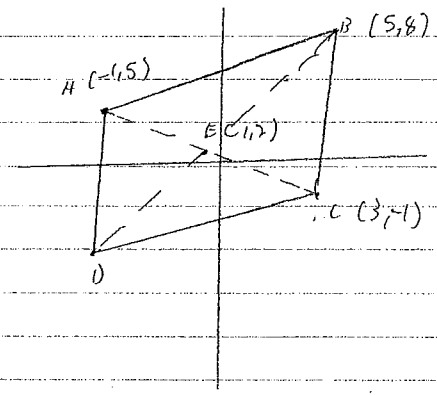
$3 = -\frac{2}{3}(2) + b$

$3 = -1\frac{1}{3} + b$ ✓

$b = 4\frac{1}{3}$ X

$\therefore y = -\frac{2}{3}x + 4\frac{1}{3}$ or $3y = -2x + 13$

or $0 = 3y + 2x - 13 = 0$



Question 2 (8)

ai $\Delta = b^2 - 4 \times 1 \times (2b+3)$

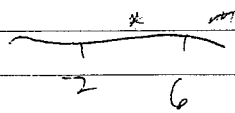
$= b^2 - 4b - 12$ ✓

$\therefore (b-6)(b+2)$

$b = 6 \quad b = 2$

$b = 6, -2$

ii $b = 6, -2$



$-2 < b < 6$ ✓

iii $\alpha\beta + 3(\alpha+\beta) = 0$

$\therefore \alpha+\beta = b$ $\quad \alpha\beta = b + 3$ ✓

$\therefore b + 3 + 3b = 0$

$\therefore 4b + 3 = 0$

$4b = -3$

$b = \frac{-3}{4}$ X

Bi $\alpha+\beta = \frac{1}{2}$ ✓

ii $\alpha\beta = \frac{-5}{2}$ ✓

iii $\alpha\beta - 2\alpha - 2\beta + 4$

$\alpha\beta - 2(\alpha+\beta) + 4$

$\therefore \frac{-5}{2} - 2(\frac{1}{2}) + 4$

$= \frac{-5}{2} - 1 + 4$

$\therefore = \frac{1}{2}$ ✓

C $u(\cos\theta(\cos\theta+1)) = 3$

$\therefore 2\cos\theta = 3 \quad 2\cos\theta + 2 = 3$

$\cos\theta = \frac{3}{2} \quad \cos\theta = \frac{1}{2}$

C $u(1-\sin^2\theta) + 2\sin\theta = 3$

$u - u\sin^2\theta + 2\sin\theta = 3$ ✓

$-u\sin^2\theta + 2\sin\theta = 1$

$-2\sin\theta(2\sin\theta - 1) = 1$

$-2\sin\theta = 1 \quad 2\sin\theta - 1 = 1$ X

$\sin\theta = -\frac{1}{2} \quad \sin\theta = 1$

$\therefore \theta = 210, 330, 90$

C $u(1-\sin^2\theta) + 2\sin\theta - 3 = 0$

$u - u\sin^2\theta + 2\sin\theta - 3 = 0$

$u\sin^2\theta - 2\sin\theta - 1 = 0$

$4u^2 - 2u - 1 = 0$

$\frac{2 \pm \sqrt{2^2 - 4(4)(-1)}}{8}$

$= \frac{2 \pm \sqrt{20}}{8} \quad \frac{2 - 2\sqrt{5}}{8}$

$= \frac{1 \pm \sqrt{5}}{4}$

but $u = \sin\theta$

$\sin\theta = \frac{1+\sqrt{5}}{4}$ $\quad \theta = \frac{1-\sqrt{5}}{4}$

$= 54, 126°, 198, 342$

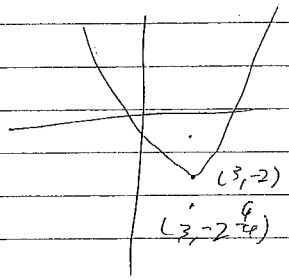
Question 3

(11)

(a) $x^2+6x+2^2+y^2+8y+4^2=-11+4+16$ ✓

$(x+2)^2+(y+4)^2=9$ ✓

$\therefore$ centre $=(-2,-4)$ ✓ radius $=3$ ✓



(b) $x^2-6x=6y+3$

$x^2-6x+3^2=6y+3+9$

$(x-3)^2=6y+12$ ✓

$(x-3)^2=6(y+2)$ ✓

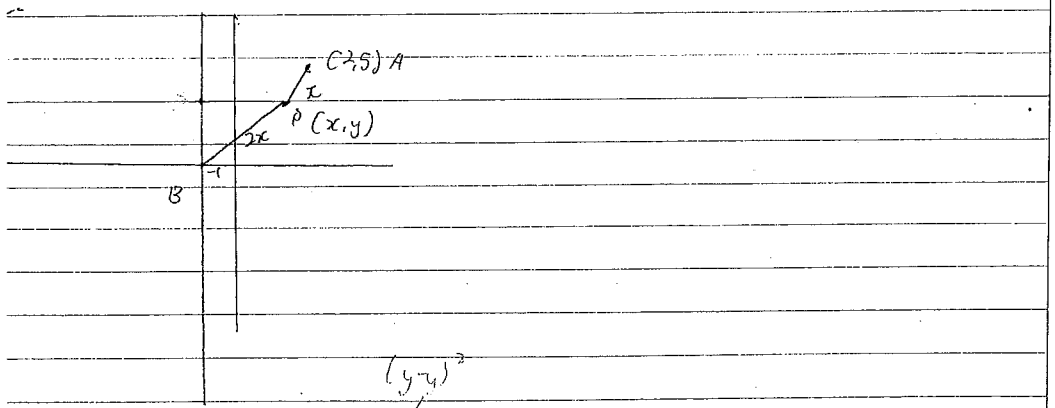
I focal length $=\frac{6}{4}$ units ✓

ii vertex $=(3,-2)$ ✓

iii ~~$(3,\frac{1}{2})$~~ $(3,-\frac{1}{2})$ ✓

v ~~$y=-2\frac{6}{4}$~~

$y=-3\frac{1}{2}$ ✓



$(y-4)^2$

$\therefore PA=2PB$ ✓

$\sqrt{(x-2)^2+(y-5)^2}=2\sqrt{(x+1)^2+(y-4)^2}$ ✗

$x^2-4x+4+y^2-10y+25=4(x^2+2x+1+y^2)$

$x^2-4x+4+y^2-10y+25=4x^2+8x+4+4y^2$ 2

$0=3x^2+12x+3y^2+10y-25$ ✗