THE SCOTS COLLEGE

2011

PRE TRIAL EXAMINATION

YEAR 12 MATHEMATICS

General Instructions

- Eight questions of equal value
- 5 minutes reading time
- Working time - 2 hours
- Write using blue or black pen
- Board approved calculators may be used
- Start a new booklet for each question
- All necessary working should be shown in every question
- Standard Integrals Table is attached

TOTAL MARKS: 80

WEIGHTING: 30 %

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Question 1: 10 Marks **START A NEW PAGE** Marks

a) Find the value of:

i. $\log_3 9$ 1

ii. $\log_e 1$ 1

b) Using $\log_a 2 = 0.387$ and $\log_a 3 = 0.613$, find the value of $\log_a 12$. 2

c) Differentiate with respect to x:

i. $x^2 e^{2x}$ 2

ii. $(e^x - e^{-x})^2$ 2

d) By first apply the log laws, differentiate $y = \ln\left(\frac{x+2}{x-1}\right)$ 2

END OF QUESTION 1

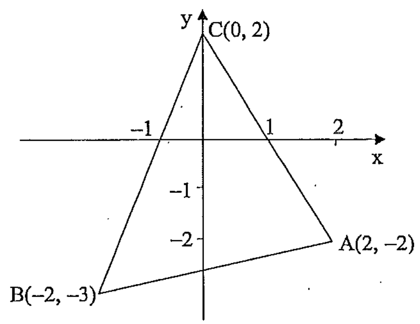
Question 2: 10 Marks **START A NEW PAGE**

a) The diagram below shows the points A(2, –2), B(–2, –3) and C(0, 2) which are the vertices of a triangle ABC. Marks

i. Show the equation of the line AC in the general form is $2x + y - 2 = 0$ 2

ii. Calculate the perpendicular distance of B from the side AC. 2

iii. Find the area of ΔABC. 3



b) For the parabola $x^2 = 16y$ state:

i. the focal length 1

ii. the coordinates of the focus 1

iii. the equation of the directrix 1

END OF QUESTION 2

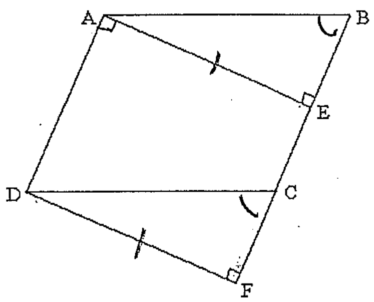
Question 3: 10 Marks **START A NEW PAGE** Marks

a)

i. Write down the discriminant of $7x^2 + 5x + k$. 1

ii. For what values of k does $7x^2 + 5x + k = 0$ have equal roots? 1

b) ABCD is a parallelogram. $\angle AEB = \angle DFC = 90°$



i) Show that $\Delta ABE \equiv \Delta DCF$. 2

ii) Show that AEFD is a rectangle. 3

c) Solve for x: 2

$|1+2x| = 7$

d) Express $\frac{1}{4+\sqrt{3}}$ with a rational denominator. 1

END OF QUESTION 3

Question 4: 10 Marks **START A NEW PAGE**

a) For what value of x is the tangent to the curve $y = e^{3x}$ parallel to the line $y = 6x$ 3

b) Evaluate:

i. $\int_2^3 e^{2x-4} dx$. 2

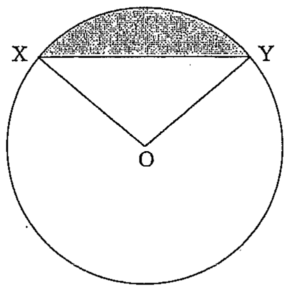
ii. $\int_1^3 \frac{x}{x^2+1} dx$. 2

c) Find $\frac{d}{dx}(\ln x)^2$ and hence evaluate $\int_1^2 \frac{\ln x}{x} dx$. 3

END OF QUESTION 4

Question 5: 10 Marks **START A NEW PAGE** Marks

a) In the diagram below, XY is an arc of a circle of radius 10 cm and $\angle XOY = \frac{3\pi}{8}$. Find the area of the shaded region correct to the nearest cm^2. 2



b) Draw a neat sketch of $y = 3\cos 2x$ for $0 \le x \le \pi$. State period, amplitude and label all x intercepts. 3

c) State the exact value of $\cos\frac{\pi}{6}$ 1

d) A straight road was constructed to cut a dangerous bend on a country road. It was found that the bend was part of an arc of radius 170 metres and the straight road was 250 metres long.

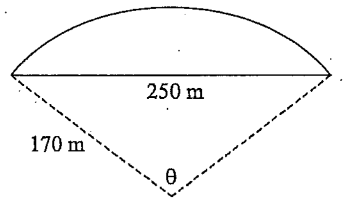


DIAGRAM NOT DRAWN TO SCALE

i. Use the cosine rule to find the size of θ correct to the nearest degree. 2

ii. Find the distance by which the old road was shortened. Answer correct to the nearest metre. 2

END OF QUESTION 5

Question 6: 10 Marks **START A NEW PAGE**

a) For the curve $y = \frac{1}{x}$ in the domain $x > 0$

i. write down the range of this function; 1

ii. use calculus to show that it:

α. has no stationary points; 2

β. is always decreasing; 1

γ. is always concave up. 2

b) Calculate the following limit:

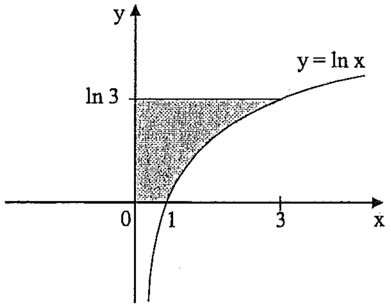
$$\lim_{x \to 3} \frac{2x^4 - 6x^3}{x^2 - 5x + 6}$$ 2

c) Find the exact area of an equilateral triangle that has side lengths 10cm. 2

END OF QUESTION 6

Question 7: 10 Marks **START A NEW PAGE** Marks

a) The diagram shows the area bounded by the graph $y = \ln x$, the co-ordinate axes and the line $y = \ln 3$.



NOT TO SCALE

i. Find the shaded area. 3

ii. Hence find the exact value of $\int_1^3 \ln x\,dx$. 1

b) Find the co-ordinates of the stationary point on the curve $y = e^{-x^2}$ and determine its nature. 3

c) Find the volume of the solid formed when the area bounded by the lines $x = 0$ and $x = 1$ and the curve $y = e^x$ is rotated about the x axis. Leave your answer in exact form. 3

END OF QUESTION 7

Question 8: 10 Marks **START A NEW PAGE** Marks

a) For the curve $y = xe^x$

i) Find any stationary points and determine their nature 2

ii) Find any asymptotes and intercepts. 2

iii) Sketch the curve 1

b) A normal is drawn to the curve $y = e^{2x}$ at the point $P(\log_e 2, 4)$.
The normal cuts the x-axis at Q which has co-ordinates $(32 + \log_e 2\,, 0)$.

i. Show that the equation of the normal at P is $x + 8y = 32 + \log_e 2$. 2

ii. Find the area of the region bounded by the curve $y = e^{2x}$, the normal at P and the co-ordinate axes. 3

END OF EXAM

SOLUTIONS

Question 1

a) i) $\log_3 9 = \frac{\log_e 9}{\log_e 3}$

$= 2$

ii) $\log_e 1 = 0$

b) $\log_a 12 = \log_a(4 \times 3)$

$= \log_a(2^2 \times 3)$

$= \log_a 2^2 + \log_a 3$

$= 2\log_a 2 + \log_a 3$

$= 2 \times 0.387 + 0.613$

$= 1.387$

c) i) $y = x^2 e^{2x}$ $\quad u = x^2 \quad v = e^{2x}$

$\frac{dy}{dx} = vu' + uv'$ $\quad u' = 2x \quad v' = 2e^{2x}$

$= e^{2x} \times 2x + x^2 \times 2e^{2x}$

$= 2xe^{2x} + 2x^2e^{2x}$

$= 2xe^{2x}(1 + x)$

ii) $y = (e^x - e^{-x})^2$

$= \left(e^x - \frac{1}{e^x}\right)^2$

$= \left(\frac{e^{2x} - 1}{e^x}\right)^2$

$= \frac{e^{4x} - 2e^{2x} + 1}{e^{2x}}$

$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$ $\quad u = e^{4x} - 2e^{2x} + 1 \quad v = e^{2x}$

$u' = 4e^{4x} - 4e^{2x} \quad v' = 2e^{2x}$

$= \frac{e^{2x} \times (4e^{4x} - 4e^{2x}) - (e^{4x} - 2e^{2x} + 1) \times 2e^{2x}}{(e^{2x})^2}$

$= \frac{4e^{4x}(e^{2x} - 1) - 2e^{2x}(e^{2x} - 1)^2}{e^{4x}}$

$= \frac{e^{2x}(e^{2x} - 1)[4e^{2x} - 2(e^{2x} - 1)]}{e^{4x}}$

over the

$= \frac{e^{2x}(e^{2x} - 1)[4e^{2x} - 2e^{2x} + 2]}{e^{4x}}$

$= \frac{(e^{2x} - 1)(2e^{2x} + 2)}{e^{2x}}$

$= \frac{2(e^{2x} - 1)(e^{2x} + 1)}{e^{2x}}$

$= \frac{2(e^{4x} - 1)}{e^{2x}}$

d) $y = \ln\left[\frac{x + 2}{x - 1}\right]$

$= \ln(x + 2) - \ln(x - 1)$

$\frac{dy}{dx} = \frac{1}{x + 2} - \frac{1}{x - 1}$

$= \frac{x - 1 - (x + 2)}{(x + 2)(x - 1)}$

$= \frac{x - 1 - x - 2}{x^2 - x + 2x - 2}$

$= \frac{-3}{x^2 - x - 2}$

$= \frac{-3}{(x + 2)(x - 1)}$

Question 2

a) i) m of AC $= \dfrac{y_2 - y_1}{x_2 - x_1}$ $A(2,-2)$ $C(0,2)$

$$= \frac{2 - -2}{0 - 2}$$

$$= \frac{2+2}{-2}$$

$$= \frac{4}{-2}$$

$$= -2$$

Equation of line AC is:

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -2(x - 0)$$

$$y - 2 = -2x$$

$$2x + y - 2 = 0$$

∴ equation of line AC in general form is

$2x + y - 2 = 0$

$2x + y - 2 = 0$ $a = 2$, $b = 1$, $c = -2$

ii) $D\perp = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$ $B(-2,-3)$

$$= \frac{|2 \times -2 + 1 \times -3 + -2|}{\sqrt{2^2 + 1^2}}$$

$$= \frac{|-4 - 3 - 2|}{\sqrt{4+1}}$$

$$= \frac{|-9|}{\sqrt{5}}$$

$= \dfrac{9}{\sqrt{5}}$ units   or $\dfrac{9\sqrt{5}}{5}$ units

iii) Area of $\Delta ABC = \frac{1}{2} \times$ base $\times$ height

$$= \frac{1}{2} \times \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \times \frac{9\sqrt{5}}{5}$$

$$= \frac{9\sqrt{5}}{10} \times \sqrt{(2-0)^2 + (-2 - -2)^2}$$

$$= \frac{9\sqrt{5}}{10} \times \sqrt{(2^2) + (0)^2}$$

$$= \frac{9\sqrt{5}}{10} \times \sqrt{4}$$

$$= \frac{9\sqrt{5}}{10} \times 2$$

$$= \frac{18\sqrt{5}}{10}$$

$= \dfrac{9\sqrt{5}}{2}$ units^2

b) i) $x^2 = 16y$ $x^2 = 4ay$

$$4a = 16$$

$$a = \frac{16}{4}$$

$$\therefore a = 4$$

∴ focal length $= a$

$= 4$

ii) focus S is $(0, a) = (0, 4)$

∴ focus $(0, 4)$

iii) directrix is $y = -a$

∴ directrix $y = -4$

Question 3

a) i) $7x^2+5x+k$

discriminant $\Delta = b^2-4ac$

$= 5^2 - 4\times 7\times k$

$= 25-28k$ ✓

ii) for $7x^2+5x+k=0$ to have equal roots

$\Delta = 0$

$\Delta = 25-28k$

$0 = 25-28k$

$28k = 25$

$k = \frac{25}{28}$ ✓

$\therefore$ for $7x^2+5x+k=0$ to have equal roots $k=\frac{25}{28}$

b) i) In $\triangle ABE$ & $\triangle DCF$

$AB = DC$ ✓ (opp sides of parallelogram $=$)

$\angle ABE = \angle DCF$ (transversal FB passes through $\parallel$ lines AB & DC $\therefore$ corresponding $\angle$'s on $\parallel$ lines $=$)

$\angle AEB = \angle DCF$ (Both $= 90°$ given)

$\therefore \triangle ABE \equiv \triangle DCF$ (ASA)

ii) $AE = DF$ (corresponding sides in $\equiv \triangle$'s $=$)

$BE = CF$ (corresponding sides in $\equiv \triangle$'s $=$)

Let $BE = x$ $\therefore CF = x$, let $EC = y$

$BE + EC = BC$

$BC = AD$ (opp sides of parallelogram $=$)

$\therefore AD = x+y$

$CF + EC = EF$

$= x+y$

however AD also $= x+y$

$\therefore EF = AD$

$\angle AEF = 180° - 90°$ ($\angle$ sum of straight line $= 180°$)

$= 90°$

$BC \parallel AD$ (opp sides in parallelogram $\parallel$)

$\therefore BF \parallel AD$ (BC lies on line BF)

$\therefore \angle ADF = 180° - 90°$ (co-interior $\angle$'s $= 180°$)

$= 90°$

$\angle DAE = 360° - 90° - 90° - 90°$ ($\angle$ sum of ~~quadrilatera~~ 4 sided polygon $= 360°$)

$= 90°$

$\therefore$ AEFD is a rectangle (opp sides $=$ as shown above
All $\angle$'s $= 90°$ as shown above)

c) $|1+2x|=7$

$\therefore\ 1+2x=7$ or $1+2x=-7$

$2x=7-1 \qquad 2x=-7-1$

$x=\frac{6}{2} \qquad x=\frac{-8}{2}$

$x=3 \qquad x=-4$

$\therefore x=3$ or $x=-4$

d) $\frac{1}{4+\sqrt{3}} = \frac{1}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}}$

$= \frac{4-\sqrt{3}}{4^2-(\sqrt{3})^2}$

$= \frac{4-\sqrt{3}}{16-3}$

$= \frac{4-\sqrt{3}}{13}$

Question 4

a) $y=6x$ ①

m_1 of ① is 6

$y=e^{3x}$ ②

$\frac{dy}{dx}=3e^{3x}$

for the tangent at the curve $y=e^{3x}$ to be parrallel to $y=6x$ m_1 must equal m_2

If $m_1=m_2$

$6=\frac{dy}{dx}$

$6=3e^{3x}$

$\frac{6}{3}=e^{3x}$

$2=e^{3x}$

$\log_e 2=3x$

$x=\frac{\ln 2}{3}$

$\therefore x=\frac{\ln 2}{3}$ for tangent at $y=e^{3x}$ to be parrallel to $y=6x$

b) i) $\int_2^3 e^{2x-4}\,dx = \int_2^3 e^{2x}\cdot e^{-4}\,dx$

$= e^{-4}\int_2^3 e^{2x}\,dx$

$= e^{-4}\left[\frac{1}{2}e^{2x}\right]_2^3$

$= \frac{1}{2}e^{-4}\left[e^{2x}\right]_2^3$

$= \frac{1}{2}e^{-4}\left[e^{2\times 3}-e^{2\times 2}\right]$

over the page

$e^6 = (e^3)^2$

$= \frac{1}{2} e^{-4} [e^6 - e^4]$

$\cancel{= \frac{1}{2} e^{-4} \times e^2 (e}$

$\frac{1}{2}$

$= \frac{1}{2} e^{-4} (e^6 - e^4) = \frac{1}{2}(e^2 - 1)$

× Please simplify

ii) $\int_1^3 \frac{x}{x^2+1} dx = \int_1^3 \frac{x}{x^2+1} \times \frac{2}{2} dx$

$= \frac{1}{2} \int_1^3 \frac{2x}{x^2+1} dx$

$= \frac{1}{2} \left[\ln(x^2+1) \right]_1^3$

$= \frac{1}{2} [\ln(3^2+1) - \ln(1^2+1)]$

$= \frac{1}{2} (\ln 10 - \ln 2)$

$= \frac{1}{2} (\ln(5 \times 2) - \ln 2)$

$= \frac{1}{2} (\ln 5 + \ln 2 - \ln 2)$

$= \frac{1}{2} (\ln 5 + 0)$

$= \frac{1}{2} \ln 5$

$= \frac{\ln 5}{2}$ ✓

c) $y = (\ln x)^2$ Let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$y = u^2$

$\frac{dy}{du} = 2u$

$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$

$= \frac{1}{x} \times 2u$

$= \frac{1}{x} \times 2\ln x$

$= \frac{2\ln x}{x}$ ✓

$\int_1^2 \frac{\ln x}{x} dx = \int_1^2 \frac{\ln x}{x} \times \frac{2}{2} dx$

$= \frac{1}{2} \int_1^2 \frac{2\ln x}{x} dx$ ✓

$= \frac{1}{2} \left[(\ln x)^2 \right]_1^2$

$= \frac{1}{2} [(\ln 2)^2 - (\ln 1)^2]$

$= \frac{1}{2} [(\ln 2)^2 - 0]$

$= \frac{1}{2} (\ln 2)^2$

$= \frac{(\ln 2)^2}{2}$ ✓

Question 5

a) Area of sector $= \frac{\cancel{\theta}}{\cancel{360}} \cdot \frac{\theta}{2\pi} \times \pi r^2$

$= \frac{\theta}{2} r^2$

$= \frac{\frac{3\pi}{8}}{2} \times 10^2 \qquad \theta = \frac{3\pi}{8},\ r = 10$

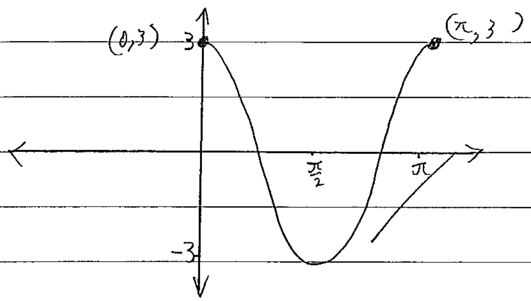
$= \frac{3\pi}{16} \times 100$

$= \frac{300\pi}{16}$

$\doteq 58.904\ldots$

$\doteq 59 \text{ cm}^2$

b)



period $= \frac{2\pi}{2}$

$= \pi$

$\therefore$ period $= \pi$ Amplitude $= 3$

c) $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

d) i) $a^2 = b^2 + c^2 - 2bc\cos A$

$2bc\cos A = b^2 + c^2 - a^2$

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$

$= \cos^{-1}\left(\frac{170^2 + 170^2 - 250^2}{2 \times 170 \times 170}\right)$

$= \cos^{-1}(-0.0813148\ldots)$

$= 94°39'50.94''$

$\doteq 95°$

ii) arc length $= 2\pi r \times \frac{\theta}{360}$

$= 2\pi r \times \frac{95°}{360°}$

$= 2 \times \pi \times 170 \times \frac{95°}{360°}$

$\doteq 280.874\ldots$ m

Distance old road was shortened:

= Distance old road − distance new road

$= 280.874 - 250$

$\doteq 30.87418\ldots$ m

$\doteq 31$ m

$\therefore$ old road was shortened by 31 m

Question 6

a) $y = \frac{1}{x}, x > 0$

i) R: $y \neq 0$ as $0 \neq \frac{1}{x}$ $\therefore x \neq \frac{1}{0}$, undefined

$y > 0$

ii) a) $y = \frac{1}{x}$

$y = x^{-1}$

$\frac{dy}{dx} = -x^{-2}$

when $\frac{dy}{dx} = 0$

$0 = -x^{-2}$

$0 = -\frac{1}{x^2}$

$x = \sqrt{-\frac{1}{0}}$, undefined

$\therefore$ no stationary points

β) $y = \frac{1}{x}$, $\frac{dy}{dx} = -x^{-2}$

$\frac{d^2y}{dx^2} = - -2x^{-3}$

$= 2x^{-3}$

$\therefore y'' = \frac{2}{x^3}$ $\therefore$ for all $x > 0$, y'' will get smaller as the denominator x^3 gets bigger

ie. $y'' = \frac{2}{x^3}$ $\quad y'' = \frac{2}{(1)^3} = 2$

$\therefore y = \frac{1}{x}$ is always decreasing $\quad y'' = \frac{2}{(2)^3} = 0.25$

as $y'' \to 0$ as $x \to \infty$ in the $\quad y'' = \frac{2}{\ldots} = 0.07$

γ) for a function to be concave up

$y'' > 0$

In $y = \frac{1}{x}$

$y'' = \frac{2}{x^3}$

as $x \to \infty$, $y'' \to 0$

since $x > 0$, y will always be ~~> 0~~

$\therefore y'' > 0$ for all x in the domain

$\therefore y = \frac{1}{x}$ is always concave up.

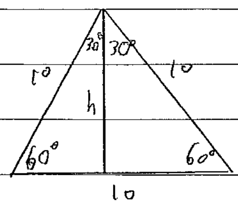
b) $\lim_{x \to 3} \frac{2x^4 - 6x^3}{x^2 - 5x + 6} = \lim_{x \to 3} \frac{\frac{2x^4}{x^4} - \frac{6x^3}{x^4}}{\frac{x^2}{x^4} - \frac{5x}{x^4} + \frac{6}{x^4}}$ (divide by the ~~highest power~~ factorise.

$= \lim_{x \to 3} \frac{2x^3(x-3)}{(x-2)(x-3)}$ $\quad = \lim_{x \to 3} \frac{2 - \frac{6}{x}}{\frac{1}{x^2} - \frac{5}{x^3} + \frac{6}{x^4}}$

$= \frac{2(3^3)}{3-2}$ $\quad = 2$

$= 54$

c)



$\sin 60° = \frac{h}{10}$

$h = 10 \sin 60°$

$= 10 \times \frac{\sqrt{3}}{2}$

$= 5\sqrt{3}$

$\therefore$ Area of equilateral Δ is ~~$\frac{1}{2}b$~~

$A = \frac{1}{2} b h$

$= \frac{1}{2} \times 10 \times 5\sqrt{3}$ $\quad = 25\sqrt{3}$ cm²

Question 7

a) i) $y = \ln x$

$x = e^y$

$\therefore$ shaded area $= \int_0^{\ln 3} e^y \, dy$

$= \left[e^y \right]_0^{\ln 3}$

$= \left[e^{\ln 3} - e^0 \right]$

$= [3 - 1]$

$= 2$ units^2

ii) $\int_1^3 \ln x \, dx$ = Area rectangle $(\ln 3 \times 3)$ − Shaded area

$= \ln 3 \times 3 - 2$

$= 3\ln 3 - 2$

b) $y = e^{-x^2}$ ①

$\frac{dy}{dx} = -2x \times e^{-x^2}$

$= \frac{-2x}{e^{x^2}}$

Stationary points when $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{e^{x^2}}$

$0 = \frac{-2x}{e^{x^2}}$

$0 = -2x$

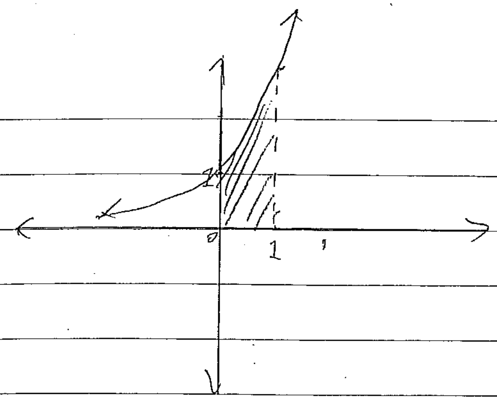
$x = 0$ sub $x = 0$ into ①

$y = e^{-0^2} = 1$

x	0^-	0	0^+
$\frac{dy}{dx}$	+ve	0	−ve
	/	−	\

$\therefore$ local max at $(0, 1)$

c)



$y = e^x$

$V = \pi \int_0^1 (e^x)^2 \, dx$

$= \pi \int_0^1 e^{2x} \, dx$

$= \pi \int_0^1 e^{2x} \, dx$

$= \pi \left[\frac{1}{2} e^{2x} \right]_0^1$

$= \pi \left(\frac{1}{2} e^{2\times 1} - \frac{1}{2} e^{2\times 0} \right]$

$= \pi \left(\frac{1}{2} e^2 - \frac{1}{2} \right)$

$= \frac{\pi}{2} (e^2 - 1)$ units^3

Question 8

a) $y = xe^{x}$

i) $y = xe^{x}$ (1) $\quad u = x \quad v = e^{x}$

$\frac{dy}{dx} = vu' + uv' \quad u' = 1 \quad v' = e^{x}$

$= e^{x} \times 1 + x \times e^{x}$

$= e^{x} + xe^{x}$

$= e^{x}(1+x)$ (2)

$\frac{d^2y}{dx^2} = e^{x} + vu' + uv'$

$= e^{x} + e^{x} + xe^{x}$

$= 2e^{x} + \cancel{x}e^{x}$

$= e^{x}(2+x)$ (3)

~~when~~ stationary points when $\frac{dy}{dx} = 0$

when $\frac{d^2y}{dx^2} = 0$

$0 = e^{x}(1+x)$

$\therefore e^{x} = 0$ or $(1+x) = 0$

$\log_e 0 = x$ $\quad x = -1$ sub into (1)

undefined, $\therefore$ $\quad y = -1 \times e^{-1}$

reject $\quad = -e^{-1}$

$= \frac{-1}{e}$

$0 = e^{x}(2+x)$

$\therefore 2+x = 0$

$x = -2$

$\therefore$ p.o.i $x = -2$

$y = -2e^{-2x}$

$\therefore$ at $(-1, -\frac{1}{e})$

x	-1^-	-1	-1^+
$\frac{dy}{dx}$	-ve	0	+ve
	\	—	/

x			
$\frac{dy}{dx}$			

$\therefore$ local min t.p. at $(-1, -\frac{1}{e})$

ii) $y = xe^{x}$, as $x \to \infty$ $y \to 0$

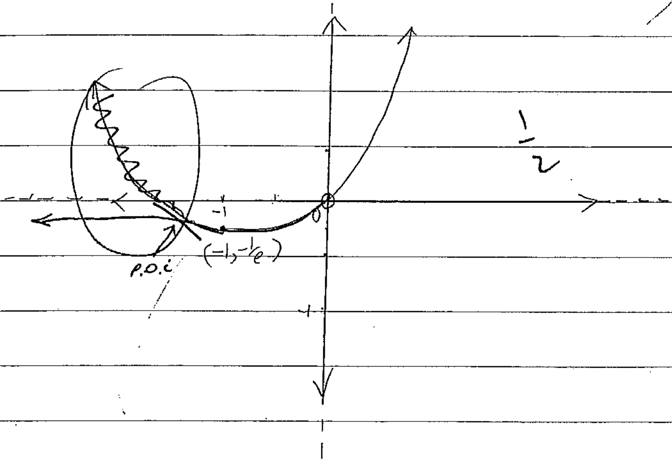
$\therefore$ asymtote at $y = 0$

when $x = 0$, $y = 0 \times e^{0}$ $\therefore$ y-intercept at $(0,0)$

$= 0 \times 1$

$= 0$

iii)



$\frac{1}{2}$

b) $y = e^{2x}$, $P(\log_e 2, 4)$

~~ii) $y = e^{2x}$~~

~~$\frac{dy}{dx} = 2e^{2x}$~~

~~when $\frac{dy}{dx} = 0$, tangent at curve is~~

~~$0 = 2e^{2x}$~~

~~$0 = e^{2x}$~~

~~1~~

i) Normal cuts point P and x-axis at Q

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0 - 4}{32 + \ln 2 - \ln 2}$

$= \frac{-4}{32}$

$= -\frac{1}{8}$

Equation of normal at P is

$y - y_1 = m(x - x_1)$

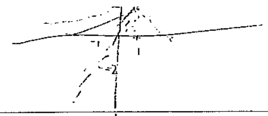
$y - 4 = -\frac{1}{8}(x - \ln 2)$

$8y - 32 = -x + \ln 2$

$x + 8y = 32 + \ln 2$

$\therefore$ equation of normal is

$x + 8y = 32 + \ln 2$



ii) Area of region bound by curve $y=e^{2x}$ and normal at P & co-ordinate axis is

$A = A_1 + A_2$

$A = \int_0^{\ln 2} e^{2x}\,dx + \frac{1}{2}bh$

$= \left[2e^{2x}\right]_0^{\ln 2} + \frac{1}{2} \times (32+\ln 2-\ln 2)\times 4)$

$= \left[2e^{2\times\ln 2} - 2e^{2\times 0}\right] + \frac{1}{2}\times 32\times 4$

$= 2\times 4 - 2\times 1 + 64$

$= 8 - 2 + 64$

$= 70$ units^2