Chapter 7

# Reasoning in Geometry 1

| Student Name | Class | Score |
|---|---|---|
| Parent Signature | Date | |

## 7:01 | Adjacent angles

Outcome SGS 4.2

If adjacent angles make a right angle, then the sum of these angles is 90°. Complementary means add to 90°.

If adjacent angles make a straight angle, then the sum of these angles is 180°.

When explaining reasons in geometry we use abbreviations – see the two examples.

**Example 1:**

$x = 17°$ (adj. comp. $\angle$s)

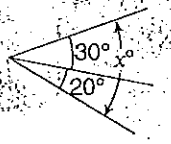

**Example 2:**

$y = 115°$ ($\angle$s make a str. $\angle$)

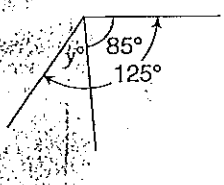



**1** Find the value of the pronumeral in each diagram.

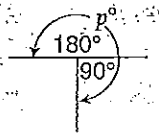
a



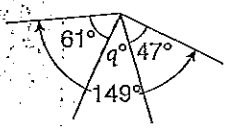

b




c

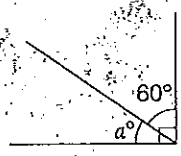



d

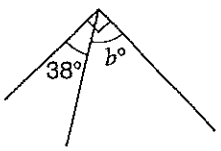



**2** Write down the value of the pronumeral in each diagram and give a geometrical reason for your answer.

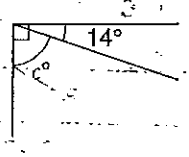
a



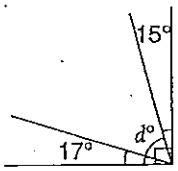

b




c

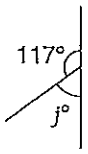



d

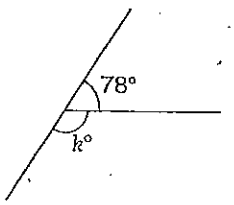



**3** Write down the value of the pronumeral in each diagram and give a geometrical reason for your answer.

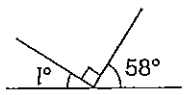
a



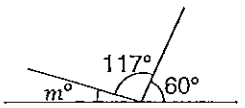

b




c




d




## 7:02 | Angles at a Point and Vertically Opposite Angles

Outcome SGS 4.2

Angles placed like a and d in this diagram are called vertically opposite angles.

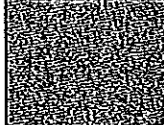
They are formed when two straight lines intersect. Vertically opposite angles are always equal in size.

**Example 1:** What is the size of the angle marked x?


$x = 98°$ (vert. opp. $\angle$s)

A group of adjacent angles at a point add to 360°. This is because there are 360° in a full circle (or one complete revolution).

**Example 2:** What is the size of the angle marked x?



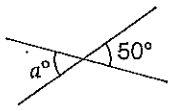

$x + 90° + 160° = 360°$

$x + 250° = 360°$

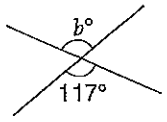
$x = 360° - 250° = 110°$ ($\angle$s at a point)

**1** Write down the value of the pronumeral in each diagram and give a geometrical reason for your answer. All the lines that cross at a point are straight in this question.

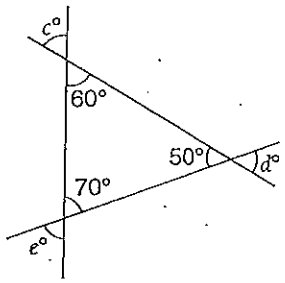
a



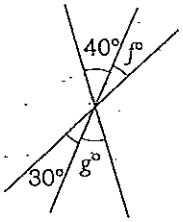

b



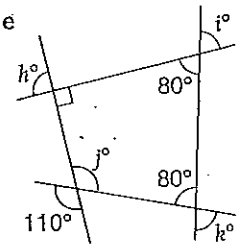

c



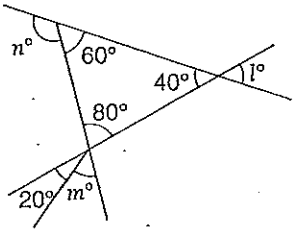

d



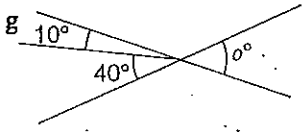

e



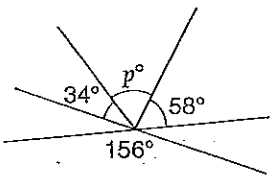

f




g

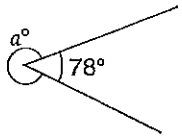



h

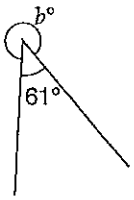



**2** Write down the value of the pronumeral in each diagram and give a geometrical reason for your answer.

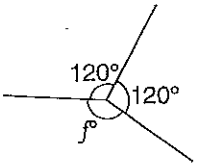
a



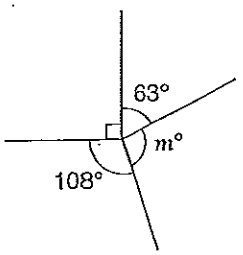

b



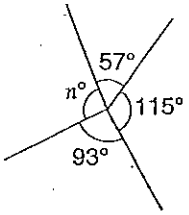

c



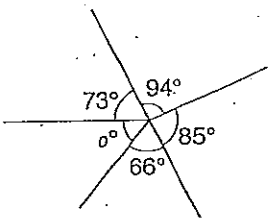

d




e

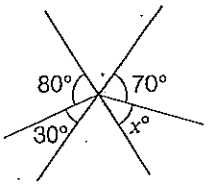



f

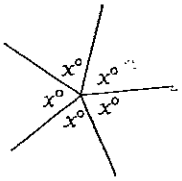



**3** Write down the value of the pronumeral in each diagram and give a geometrical reason for your answer.

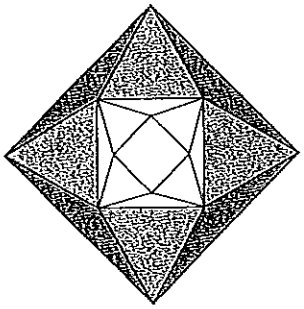
a




b




## Fun Spot




**1** How many right-angles are there in this shape? Don't count ones that have been split. _____

**2** How many obtuse ones are there? Again, don't count split ones. _____

| Student Name | Class | Score |
|---|---|---|
| Parent Signature | Date | |

# 7:03 Angles Associated with Parallel Lines

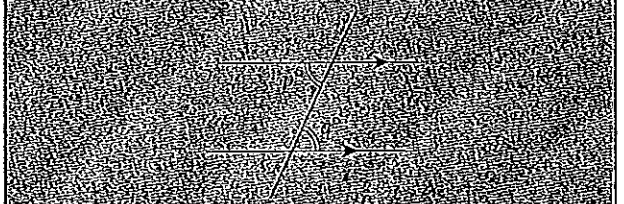
Outcome SGS 4.2

When a transversal crosses a pair of parallel lines there are several different angle relationships.

**Alternate angles** are between the parallel lines and on opposite sides of the transversal.

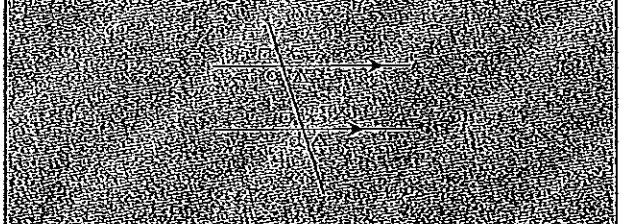
Alternate angles on parallel lines are equal.

The angles marked $p$ and $q$ are alternate angles.



**Corresponding angles** are both above or both below the parallel lines and are on the same side of the transversal. Corresponding angles on parallel lines are equal.

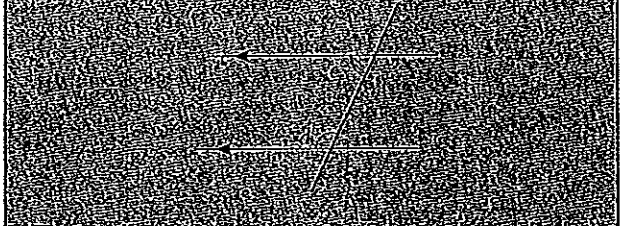
This diagram shows the four pairs of corresponding angles.



**Co-interior angles** are between the parallel lines and on the same side of the transversal.

Co-interior angles on parallel lines are supplementary (add to 180°).

The angles marked $c$ and $x$ are co-interior angles.

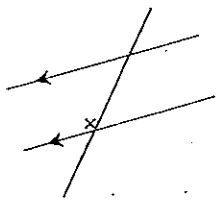


**Example:** What is the size of the angle marked $x$?

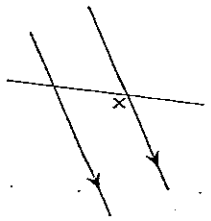

**Answer:** $x = 180° - 121° = 59°$ (co-int. $\angle$s, $\parallel$ lines)

**1** Mark in the angle which is alternate to the one marked × in each diagram.

a

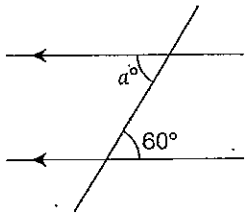


b

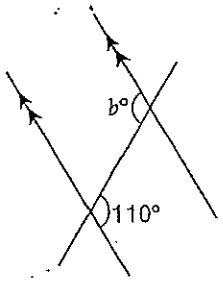


**2** In each diagram, work out the size of the unknown marked angles. Give a reason for each answer.

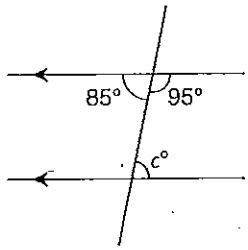
a



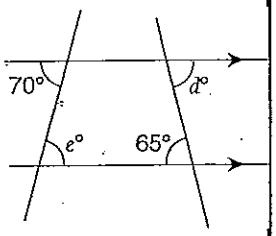

b



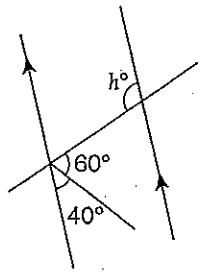

c



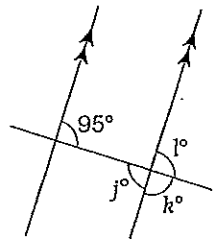

d



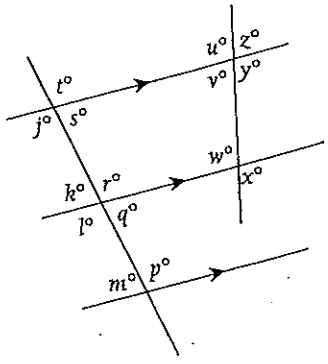

e




f




**3**



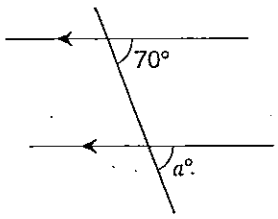
a Copy and complete this table to show pairs of corresponding angles.

| angle | is corresponding to | angle |
|---|---|---|
| $j$ | | |
| $u$ | | |
| | | $s$ |
| $x$ | | |
| | | $k$ |

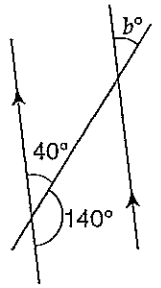
b Are angles $p$ and $t$ corresponding? _____

**4** In each diagram work out the size of the unknown marked angles. Give a reason for each answer.

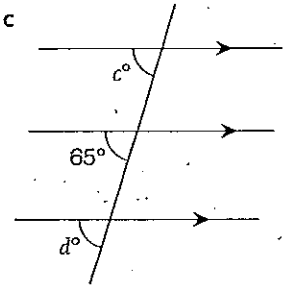
a



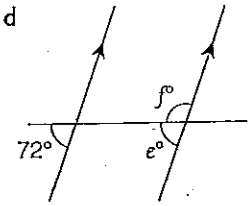

b



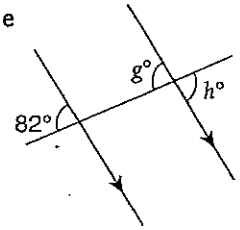

c



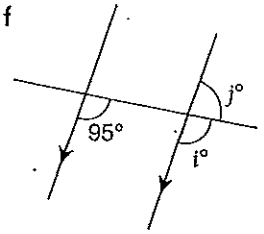

d



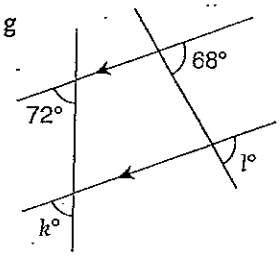

e



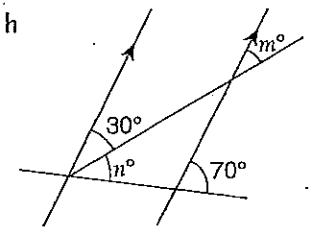

f




g

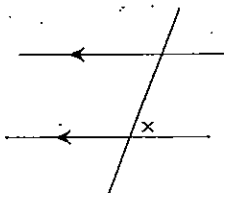



h

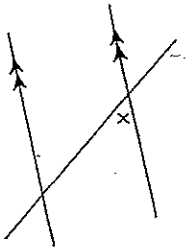



**5** Use the symbol • to mark in the angle which is co-interior to the one marked × in each diagram.

a

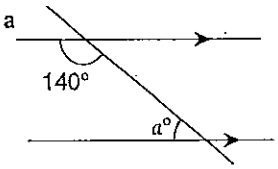



b

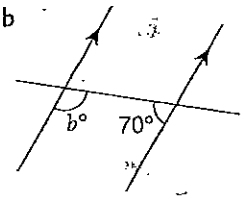



**6** In each diagram, work out the size of the unknown marked angles. Give a reason for each answer.

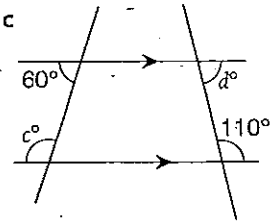
a



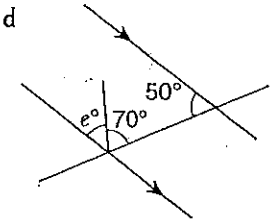

b




c



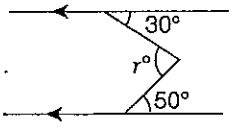

d




## Fun Spot

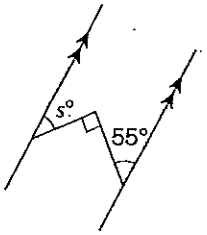
Calculate angles $r$ and $s$. *Hint*: add an extra line in each diagram.


**1**




$r =$ _____

**2**




$s =$ _____

| Student Name | Class | Score |
|---|---|---|
| Parent Signature | Date | |

# 7:04 Angle Sum of a Triangle

Outcome SGS 4.3

**The angles in a triangle add up to 180°.**

**Example 1:** Work out the size of the angle marked $q$.


**Answer:**

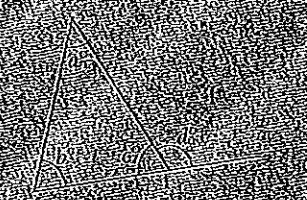
$q + 40 + 85 = 180$ (angle sum of $\triangle$)

$q + 125 = 180$

$q = 180 - 125$

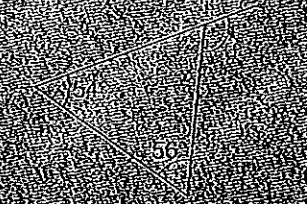
$q = 55$ (angle sum of $\triangle$)

**The exterior angle of a triangle is equal to the sum of the interior opposite angles.**




**The result means that in this triangle $d = a + b$.**

**Example 2:** Calculate the size of angle $x$.

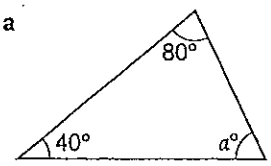



**Answer:** $x = 54 + 56$ (ext $\angle$ of $\triangle$)

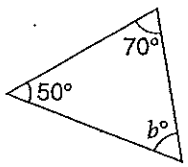
$= 110°$

**1** For each diagram work out the size of the pronumeral. Give a reason for each answer.

a

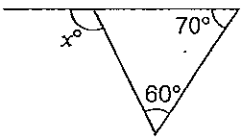



b

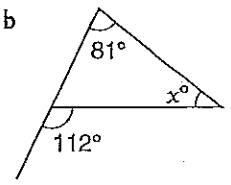



**2** For each diagram work out the size of the pronumeral. Give a reason for each answer.

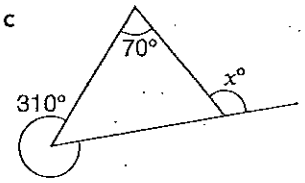
a



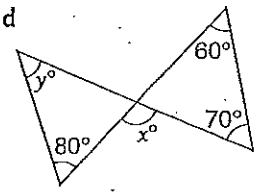

b




c

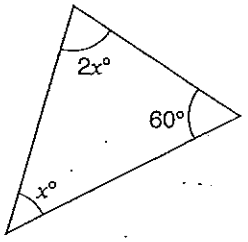



d

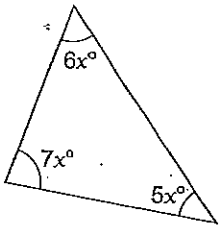



**3** Write an equation using the information in each diagram and solve it to work out the value of the pronumeral.

a

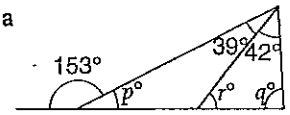



b

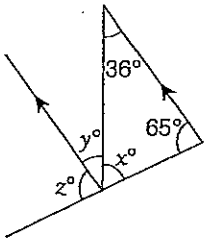



**4** Calculate the sizes of the marked angles (no reasons needed).

a




b



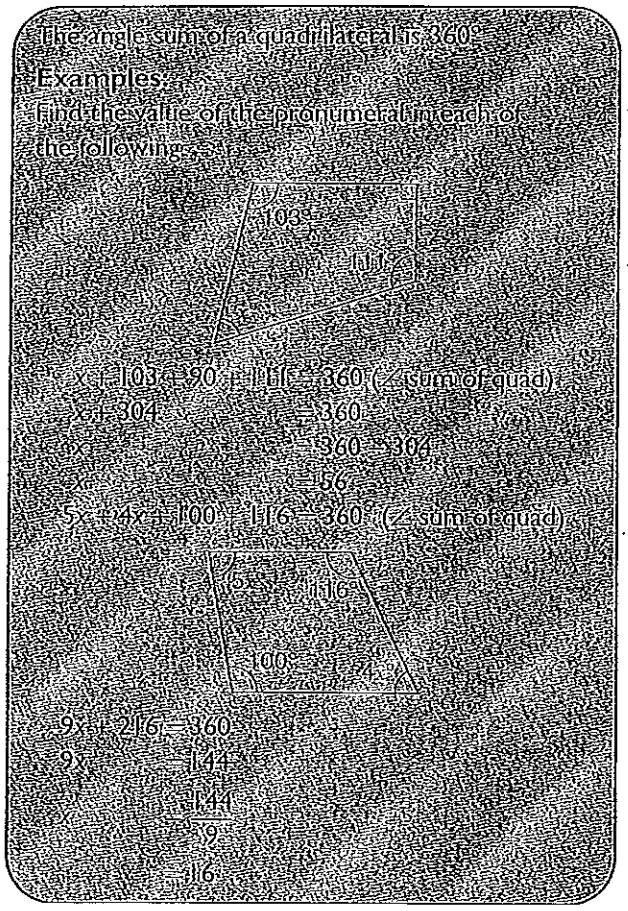
# 7:05 | Angle Sum of a Quadrilateral

Outcome SGS 4-3

The angle sum of a quadrilateral is 360°.

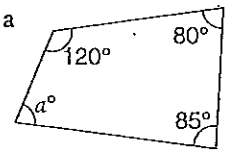
**Examples:**

Find the value of the pronumeral in each of the following.

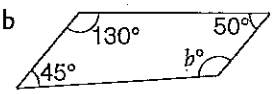



**1** Find the value of the pronumeral in each of the following. Give a reason in each case.

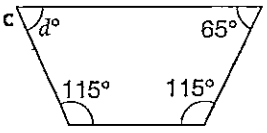
a



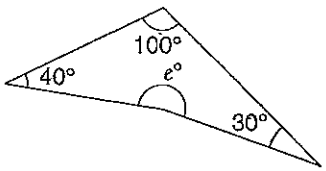

b




c

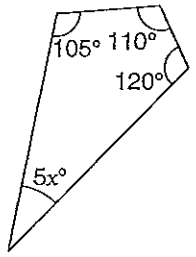



d

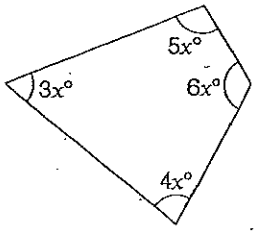



**2** Write an equation for the angle sum of each of the following and then solve it to find the value of the pronumeral.

a

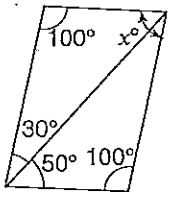



b

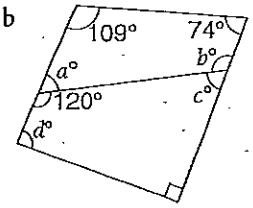



**3** Calculate the sizes of the marked angles. No reasons are needed.

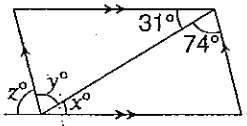
a



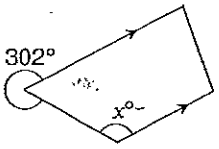

b



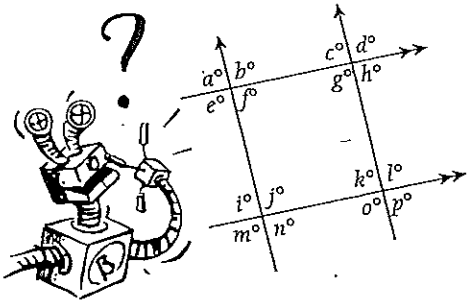

c




d




## Fun Spot





**1** How many different pairs of alternate angles are there in this diagram? _____

**2** How many different pairs of corresponding angles are there in this diagram? _____

**3** How many different pairs of co-interior angles are there in this diagram? _____

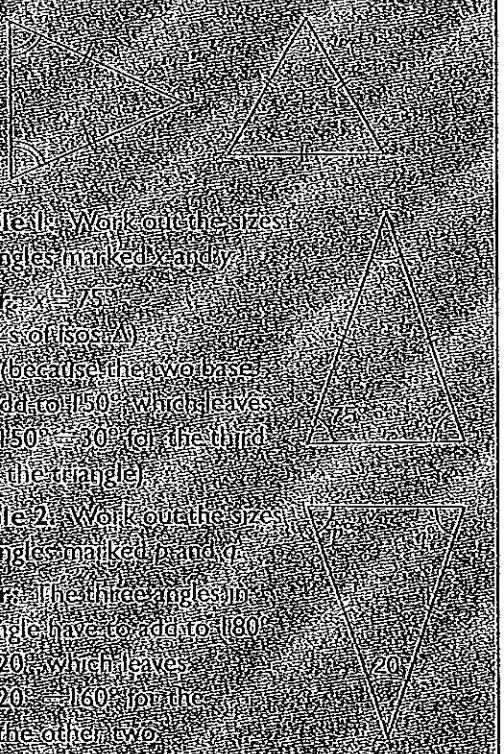
| Student Name | Class | Score |
|---|---|---|
| Parent Signature | Date | |

## 7:06 | Isosceles and Equilateral Triangles

Outcomes SGS 4.2–4.3

Isosceles triangles have:
- a pair of equal-sized angles
- two sides that are the same length.

Equilateral triangles have all sides the same length. Each angle is 60°.



**Example 1:** Work out the sizes of the angles marked $x$ and $y$.

**Answer:** $x = 75$ (equal $\angle$s of isos $\triangle$)

$y = 30$ (because the two base angles add to 150°, which leaves $180 - 150 = 30$° for the third angle of the triangle)

**Example 2:** Work out the sizes of the angles marked $p$ and $q$.

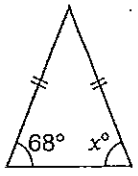
**Answer:** The three angles in the triangle have to add to 180°. One is 20°, which leaves $180 - 20 = 160$° for the sum of the other two.

The angles $p$ and $q$ are equal, so they must be $160 \div 2 = 80$°.

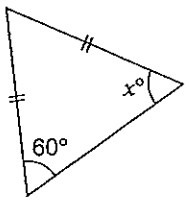
$(p = 80, q = 80)$

**1** Find the values of the pronumerals. Give reasons for all answers.

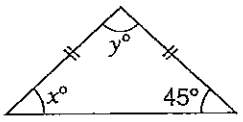
a



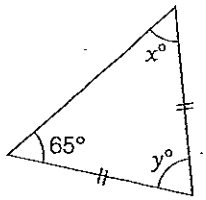

b




c

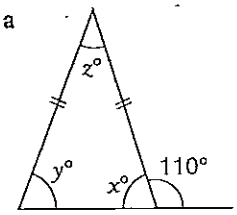



d

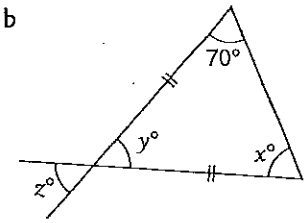



**2** Find the values of the pronumerals. Give reasons for all answers.

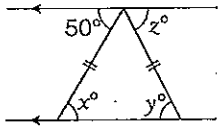
a



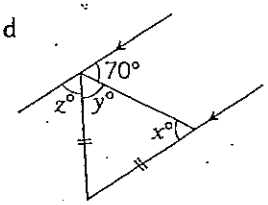

b



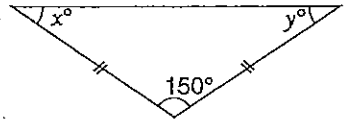

c



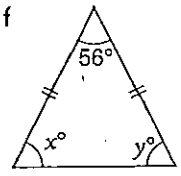

d



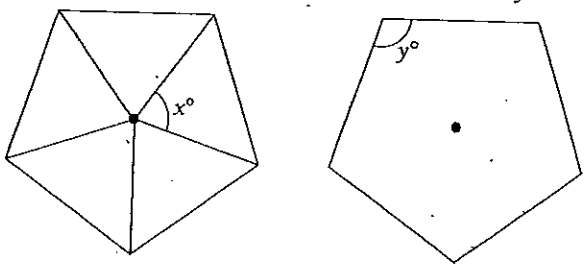

e




f



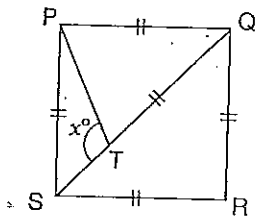
**3** Here are two views of the same regular pentagon. One view shows how the pentagon can be split up into five identical isosceles triangles that join up at the centre of the pentagon. The other view shows an *interior* angle of the pentagon, labelled *y*.




a Write down a mathematical calculation to show why angle *x* is 72°. _____

b Use the properties of isosceles triangles to help you work out the size of angle *y*. _____

**4** PQRS is a square. The length of PQ = the length of QT. Calculate the size of the angle marked *x*.



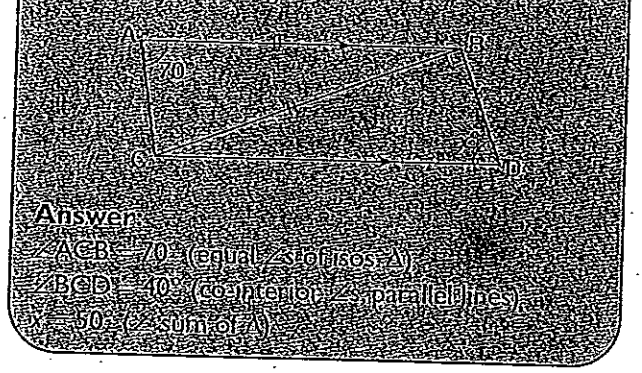

_____

## 7:07 | More Involved Numerical Problems

Outcomes SGS4.2, SGS4.3

Some harder problems involve more than one step of reasoning.

**Example:** Find the value of *x*, giving reasons for each step of working.




**Answer:**

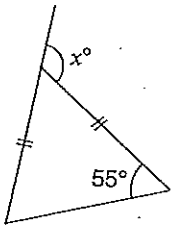
∠ACB = 70° (equal ∠s of isos △ABC)

∠BCD = 40° (co-interior ∠s, parallel lines)

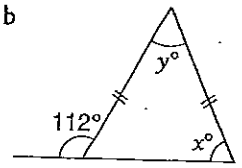
*x* = 50° (∠ sum of △)

**1** For the following diagrams, work out the sizes of the pronumerals.

a

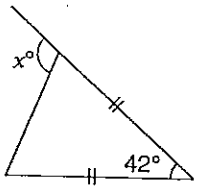



b

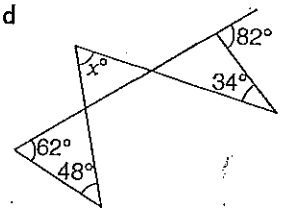



_____ _____

c



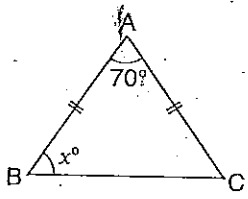

d



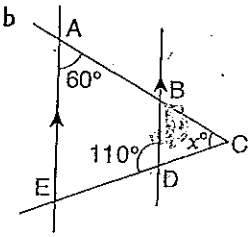

_____ _____

**2** For the following diagrams, work out the sizes of the pronumerals. Give a reason for each answer.

a




b

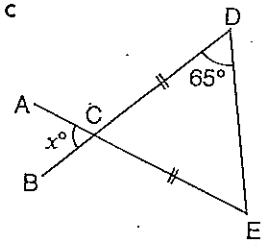



_____ _____

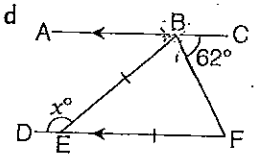
_____ _____

_____ _____

c




d

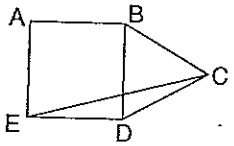



_____ _____

_____ _____

_____ _____

**3** BCD is an equilateral triangle and ABDE is a square. Calculate the sizes of these angles.




a ∠BDC _____

b. ∠CDE _____

c ∠CED _____

d ∠CEA _____