Patterns and Algebra 1

Student Name	Class	Score
Parent Signature	Date	

5:01 | Patterns and Rules Review

Outcomes PAS 4.1–4.2

1 This pattern is made up of matches and dots.

a Draw the next shape in the pattern.

b Copy and complete this table to show how the pattern continues.

Number of dots d	1	2	3	4	5	6
Number of matches m	3					

c How many matches would there be if there were:

  i 8 dots? _____

  ii 10 dots? _____

  iii 100 dots? _____

d Explain in words what you do to the number of dots to work out the number of matches.

_____

_____

_____

2 Use the rule $y = 3x - 1$ to complete the table.

x	1	2	3	4	5
y					

3 Use each table below to complete the rule underneath.

a

x	5	6	7	8
y	19	23	27	31

$y = 4x -$ _____

b

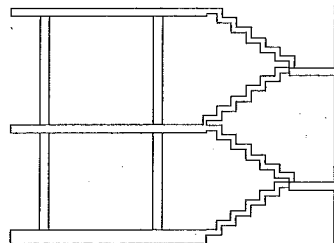| x	1	2	3	4
y	7	9	11	13

$y =$ _____ $x +$ _____

4 A construction company builds office blocks that are several storeys high. A flight of stairs joins one storey to a landing, so that two flights of stairs link one storey to another.

The diagram shows that a three-storey building would have four flights of stairs.

a How many flights of stairs would these buildings have?

  i two storeys _____

  ii four storeys _____

b Let's take n as the number of storeys. Which of these is the correct formula for f, the number of flights of stairs? _____

  i $f = n + 1$ **ii** $f = 2n - 2$

c Write down some working to show that the formula $f = 2n - 2$ gives a sensible answer when $n = 1$. Explain.

5 This diagram shows a model for an office block. The top and all four sides of the model are to be covered with square mirror tiles. One of these tiles will cover the top, and five tiles are needed for each face.

a How many tiles are needed altogether? _____

b How many tiles would be needed for a model that was twice as high, but with the same-sized top? _____

c Investigate to suggest a rule that gives the number of tiles needed when there are n tiles on one face. Write the rule:

   i in words

   _____

   ii using an algebraic formula

   _____

5:02 | Addition and Subtraction of Like Terms

Outcome PAS 4.3

> **Like terms** have the same combination of letters. Like terms can be added or subtracted.
> $$4x + 11x = 15x$$
> $$6pq - 5pq = pq$$
> $$7x + 8 - 4x - 10 = 3x - 2$$

1 Make a list of the like terms in each of the following.

a $4x$, $5y$, $6x$, $3x$, $6z$ _____

b $6p$, $2p^2$, $-p$, $6pq$ _____

c ab, $6a$, $2b$, $3ab$ _____

2 Collect like terms and simplify.

a $4x + 2x$ _____

b $x + x + x + x$ _____

c $8x + x$ _____

d $p + 2p + 4p$ _____

e $8x - 7x$ _____

f $6x - 8x$ _____

g $10p + 10p + 10p$ _____

h $x + 6x - 3x$ _____

i $x - x + x$ _____

3 Simplify the following by combining like terms.

a $2x + 4x + 5y + y$ _____

b $6x + 3 + 2x$ _____

c $8x + 2y + 6x + y$ _____

d $4x + y - 2x + 3y$ _____

e $6x + 3y - 8x$ _____

f $7x + 6y - 4x - 8y$ _____

g $4x - 8 - x - 11$ _____

h $6x - 2 + 5x + 3$ _____

i $2 - 3x + 4 + 6$ _____

j $-2x + 3 - 5x + 1$ _____

k $-2 - 6x + 1 - 6$ _____

l $5x - 3y + 2x - 6y + x$ _____

m $-11x - 7 + 8x - 5x + 1$ _____

4 Collect like terms and simplify.

a $4pq - 3pq + pq$ _____

b $6xy + xy + 2yx$ _____

c $-2cd + cd - 6dc$ _____

d $3ab - 2bc + 7ac + 7ba + 4ca - cb$ _____

_____

5 Simplify the following by combining like terms.

a $x^2 + 8x + 2x^2$

_____

_____

b $3x + 5x^2 - 11x + 2x^2 - x$
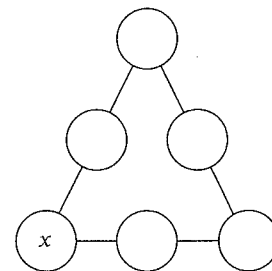
_____

_____

c $4x^2 + 5x - 3 - 5x^2 + x + 2$

_____

_____

6 Write two different algebraic expressions that could be added to give:

a $8p$ _____ _____

b $6x + 3y$ _____ _____

c $5x - 8$ _____ _____

Fun Spot

Place the five algebraic expressions

$$x - 3 \qquad x - 2 \qquad x - 1$$
$$x + 1 \qquad x + 2$$

in the circles so that each side of the triangular shape adds up to $3x$.

5:03 | Multiplication of Pronumerals

Outcome PAS 4.3

> When multiplying numbers and one or more pronumerals we show the result by writing numbers first and then pronumerals without using multiplication signs.
>
> **Examples:**
> $$4x \times -2 = -8x$$
> $$-5x \times -8y = 40xy$$
> $$3x \times 7x = 21x^2$$

1 Simplify each product.

a $8 \times 3p$ _____

b $2x \times 7$ _____

c $\frac{1}{3} \times 6p$ _____

2 Simplify these products.

a $5c \times 20d$ _____

b $6 \times 2x \times 3y$ _____

c $2x \times 3xy$ _____

d $2q \times 4p \times 8q$ _____

3 Simplify these products, which involve negative terms.

a $(-4) \times p$ _____

b $6 \times (-x)$ _____

c $(-m) \times (-n)$ _____

d $(-p) \times 2 \times (-3)$ _____

4 Simplify these products.

a $(-3) \times 2y$ _____

b $(-6x) \times (-5)$ _____

c $(-3x) \times 10y$ _____

d $(-x) \times 2y \times (-3z)$ _____

5 Complete this multiplication table.

$\times$	5	p	$3q$	$4pq$
$2p$				
$3pq$				
q				

5:04 | Division of Pronumerals

Outcome PAS 4.3

> The result of dividing is often written as a fraction.
>
> **Example:** $x \div 10y = \frac{x}{10y}$
>
> When possible, simplify a division by cancelling common factors.
>
> **Examples:**
> $$\frac{6x}{-3} = -2x$$
> $$\frac{4x}{12x} = \frac{1}{3}$$
> $$\frac{10x}{15xy} = \frac{2}{3y}$$

1 Simplify each of these divisions.

a $\frac{10p}{2}$ _____   b $\frac{4x}{6y}$ _____

c $\frac{15xy}{20x}$ _____   d $\frac{4abc}{2ac}$ _____

e $\frac{24x^2}{16x}$ _____   f $\frac{2x}{11x}$ _____

2 Write in fraction form and simplify if possible.

a $21p \div (-3)$ _____

b $(-4) \div 2x$ _____

c $(-3) \div (-15y)$ _____

d $(-1) \div 4y^2$ _____

3 Simplify these algebraic fractions by cancelling common factors.

a $\frac{18p}{-2}$ _____   b $\frac{-6q}{3}$ _____

c $\frac{-24x}{-18}$ _____   d $\frac{16x}{-12}$ _____

e $\frac{-4x}{-20}$ _____   f $\frac{60ab}{-5b}$ _____

5:05 | Multiplication and Division of Pronumerals

> Multiplication and division can both be represented in a single expression. This expression can sometimes be simplified.
>
> **Example:** $12pq \times 3 \div 4q = 36pq \div 4q$
> $= 9p$

1 Simplify these expressions.

a $6x \times 10 \div 4$ _____

b $3x \times 8 \div 2x$ _____

c $40p \div 20p \div 2$ _____

d $60pq \div 30p \times 2q$ _____

2 Write these quotients in fraction form.

a $5 \times p \div q$ _____

b $a \times 8 \div b$ _____

c $10 \times p \div q$ _____

3 Express the following as simply as possible.

a $30x \div 5$ _____

b $21p \div 14q$ _____

c $8 \times p \div 4$ _____

d $3xy \times 8x \div 18y$ _____

4 Simplify these expressions.

a $\dfrac{2x \times 6y}{3x}$ _____

b $\dfrac{3p \times 8q}{6p}$ _____

5:06 | Using Algebra

1 Write an expression for the perimeter of each of these shapes.

a

b

_____

c

d

_____

2 a Add two lines to this shape to show how it can be split into three identical parts.
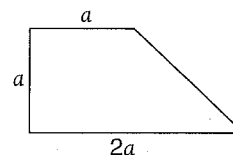
b Use the formula for the area of a triangle (area $= \frac{1}{2}$ base $\times$ height) to write down an expression for each of the parts.

_____

c Write down an expression for the area of the whole shape. _____
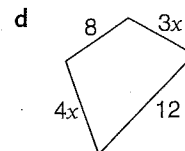
Different Stamps

investigation

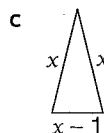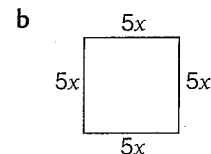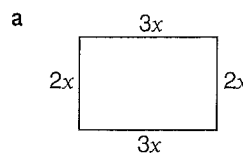You have a parcel to post to a friend in New Zealand. The cost is exactly $10. You can choose from stamps in denominations of $1, $2, $3, $4, $5, $6, $7, $8, $9 and $10. Your friend collects stamps, so you have decided to use as many *different* stamps as possible.

1 What is the greatest number of different stamps you can use? Explain.

2 Write down all the different ways in which you could use *three* different stamps.

3 If you used two different stamps, and one of the stamps was $x, write down a formula for the value of the other stamp.

4 Now work out how many different ways there are of using *any* three stamps that add to $10 (some stamps can be the same).

5 There is a process you could follow to make sure you had all the possibilities in 4. Describe it.

Patterns and Algebra 3

Student Name		Class		Score
Parent Signature		Date		

5:07 | Index Notation

Outcome PAS 4.3

You use index notation to show repeated multiplication of the same number or variable.

Examples:

$$p^4 = p \times p \times p \times p$$
$$x \times x \times x \times y \times y = x^3 y^2$$

When there are no brackets, and you have a mixture of multiplication and squaring, always do the squaring first.

Example: Work out the value of $3x^2$ if $x = 4$.

$$3x^2 = 3 \times x \times x$$
$$= 3 \times 4 \times 4$$
$$= 3 \times 16$$
$$= 48$$

1 Simplify these expressions by using index notation.

a $r \times r \times r \times r \times r$ _____

b $y \times y$ _____

c $3 \times a \times a \times b \times b$ _____

d $3 \times a \times 2 \times b$ _____

e $p \times q \times q \times q \times r \times r$ _____

f $6 \times 5 \times r \times r \times s \times s \times r$ _____

2 Write these expressions in full, using multiplication symbols.

a p^4 _____

b t^5 _____

c $6x^2$ _____

d $2xy^3$ _____

e $4x^3 y$ _____

f $20p^4 q^2$ _____

3 Simplify these by multiplying and writing in index form where possible.

a $4 \times x \times x$ _____

b $6 \times y \times 5 \times y$ _____

c $p \times 2 \times p$ _____

d $3x \times 5 \times 2x$ _____

e $2q \times 2q \times 2q$ _____

f $8x \times x$ _____

g $y \times 2y \times 3y$ _____

h $4 \times 2x \times x$ _____

i $3p \times 2q$ _____

j $5x \times y \times 3x \times 2y$ _____

4 $c = 3, d = 4, e = 8$. Work out the following.

a c^2 _____

b d^3 _____

c $(e + 1)^2$ _____

d $(d - 3)^2$ _____

e $d^2 + 5$ _____

f $(d + 5)^2$ _____

g $(c + d)^2$ _____

h $c^2 + d^2$ _____

5 $p = 2, q = 3, r = 4$. Evaluate these expressions.

a $3p^2$ _____

b $(3p)^2$ _____

c $2r^2$ _____

d $(2r)^2$ _____

e $5q^2$ _____

f $6p^2$ _____

6 $x = 1, y = 5, z = 2$. Work out the following.

a $2y^2$ _____

b $3x^2$ _____

c $4z^2$ _____

d $3y^2 + 1$ _____

e $6x^2 + y$ _____

f $7z^2 - 20$ _____

g yz^2 _____

h $2z^2 - x^2$ _____

5:08 | Grouping Symbols

Outcome PAS 4.3

> Expanding brackets means rewriting without grouping symbols. Each term inside the brackets is multiplied by the number outside.
>
> **Examples:**
> $$2(x + 3) = 2x + 6$$
> $$x(3x - 8) = 3x^2 - 8x$$
>
> Watch for multiplication by negative terms.
>
> **Example:**
> $$-7(4x - 5) = -28x + 35$$
>
> Sometimes, after expanding, an expression can be simplified by adding/subtracting like terms.
>
> **Examples:**
> $$6x + 4(2 - x) = 6x + 8 - 4x$$
> $$= 2x + 8$$
> $$7(x + 8) - 4(3 - 6x) = 7x + 56 - 12 + 24x$$
> $$= 31x + 44$$

1 Complete the gaps using the distributive law,

a $5(x + 3) = 5 \times \underline{\quad} + 5 \times \underline{\quad} = \underline{\quad} + \underline{\quad}$

b $-2(4x + 1) = -2 \times \underline{\quad} + -2 \times \underline{\quad} = \underline{\quad} - \underline{\quad}$

c $-8(3x - 1) = \underline{\quad} \times 3x - \underline{\quad} \times 1 = \underline{\quad} + \underline{\quad}$

2 Expand the brackets in these expressions.

a $6(x + 2)$ _____

b $2(x - 3)$ _____

c $5(p + q)$ _____

3 Expand each of the following.

a $6(2x + 1)$ _____

b $2(6x - 5)$ _____

c $3(10x + 8)$ _____

d $5(2 + 4x)$ _____

e $10(1 - 3x)$ _____

4 Expand.

a $x(x + 5)$ _____

b $2x(x - 3)$ _____

5 Expand the brackets in these expressions (note that negative terms are involved).

a $-4(x + 5)$ _____

b $-3(x - 6)$ _____

6 Expand the following.

a $-4(5x - 1)$ _____

b $-5(2x + 1)$ _____

c $-7(3x - 2)$ _____

d $-6(3 - 2x)$ _____

7 Expand and simplify each of these expressions.

a $5(x + 2) + 8$ _____

b $4x + 2(x + 3)$ _____

c $3x + 2(1 - 2x)$ _____

d $5(x - 4) + 6x - 7$ _____

8 Expand and simplify each of these expressions.

a $6x - 2(x + 1)$ _____

b $8x - 4(x - 4)$ _____

c $15 - 2(x + 1)$ _____

d $18x - 3(2x + 7)$ _____

e $2x - (x + 3)$ _____

f $6 - (x - 2)$ _____

g $5x - 3(x + 2)$ _____

9 Expand and simplify each of these expressions.

a $5(x + 3) + 2(x + 6)$

b $-2(3x - 1) + 3(x - 5)$

c $3(2x + 5) - 2(x + 1)$

Patterns and Algebra 4

Student Name		Class	Score
Parent Signature		Date	

5:09 | Factorising

Outcome PAS 4.3

Factorising is the reverse process of expanding.
When factorising, you are putting a set of brackets **back into** the expression.
When factorising, look for a common factor.

Example 1: Factorise $4p + 4q$.
$4p$ and $4q$ each have a common factor of 4.
This common factor is written at the front of the brackets.

Answer: $4p + 4q = 4(p + q)$

Example 2: Factorise $4x + 20$.
$4x$ and 20 each have a common factor of 4.
(*Note:* $20 = 4 \times 5$)

Answer: $4x + 20 = 4 \times x + 4 \times 5$
$= 4(x + 5)$

Example 3: Factorise $16x - 24$.
(*Note:* $16x$ and 24 have common factors of 1, 2, 4 and 8. We use the highest of these, 8, as the common factor.)

Answer: $16x - 24 = 8(2x - 3)$

Example 4: Factorise $10x + 5$.

Answer: $10x + 5 = 5(2x + 1)$
Notice how the number 1 is needed inside the brackets.

Some expressions have a pronumeral as a common factor.

Example 5: $pq - pr = p(q - r)$

Example 6: $x^2 + xy$ has a common factor of x:
$x^2 + xy = x(x + y)$

1 Write a number, letter or symbol on each line to make the factorising work correctly.

a $\quad 8x + 8y = \underline{\quad}(x + y)$

b $\quad 5p - 5q = \underline{\quad}(p - q)$

c $\quad 6x + 6y = 6(x \underline{\quad} y)$

d $\quad 4x - 4y = 4(x \underline{\quad} y)$

e $\quad 3x + 3y + 3z = 3(\underline{\quad} + y + z)$

f $\quad 7p + 7q = 7(p + \underline{\quad})$

g $\quad 2x - 2y = \underline{\quad}(x \underline{\quad} y)$

h $\quad 9p + 9q = 9(\underline{\quad} + \underline{\quad})$

2 Factorise these expressions.

a $\quad 2x + 2y$ _____

b $\quad 10p - 10q$ _____

c $\quad 5x + 5y + 5z$ _____

d $\quad 4p + 4q - 4r$ _____

3 Write a number, letter or symbol on each line to make the factorising work correctly.

a $\quad 6x + 24 = \underline{\quad}(x + 4)$

b $\quad 5x - 35 = \underline{\quad}(x - 7)$

c $\quad 2x + 18 = 2(\underline{\quad} + 9)$

d $\quad 3x + 15 = 3(x + \underline{\quad})$

e $\quad 6x - 48 = 6(x - \underline{\quad})$

f $\quad 4x - 18 = 2(2x - \underline{\quad})$

g $\quad 18x + 24 = 6(\underline{\quad} + \underline{\quad})$

h $\quad 10x - 12 = \underline{\quad}(5x - 6)$

4 Factorise these expressions.

a $\quad 2x + 10$ _____

b $\quad 3x + 12$ _____

c $\quad 5x - 15$ _____

d $\quad 4x - 24$ _____

e $\quad 8x + 16$ _____

f $\quad 30x + 60$ _____

g $\quad 9x - 63$ _____

h $\quad 2x + 58$ _____

i $\quad 9x + 12$ _____

j $\quad 4x - 6$ _____

k $\quad 3x + 3$ _____

l $\quad 30x - 18$ _____

5 Factorise these expressions.

a $\quad 18x - 30$ _____

b $\quad 20x + 4$ _____

c $\quad 13x - 26$ _____

d $\quad 12x - 3$ _____

6 Factorise these expressions.

a $\quad cx + cy$ _____

b $\quad px + qx$ _____

c $x^2 - xy$ _____

d $x^2 + 6x$ _____

e $x^2 - 12x$ _____

f $4x + xy$ _____

g $x^2 + 3xy$ _____

h $x^2 + x$ _____

5:10 | Algebraic Fractions

Outcome PAS 4.3

To add algebraic fractions, use the same method as in adding numerical fractions.

Example 1:

$$\frac{8x}{15} + \frac{x}{15} = \frac{8x + x}{15}$$
$$= \frac{9x}{15}$$
$$= \frac{3x}{5}$$

Write fractions with a common denominator before adding or subtracting.

Example 2:

$$\frac{2x}{5} - \frac{x}{3} = \frac{6x}{15} - \frac{5x}{15}$$
$$= \frac{6x - 5x}{15}$$
$$= \frac{x}{15}$$

Multiply algebraic fractions by multiplying the numerators and multiplying the denominators. Simplify your answer if possible.

Example 3:

$$\frac{2x}{3} \times \frac{5}{6x} = \frac{10x}{18x} = \frac{5}{9}$$

Divide fractions by multiplying the first fraction by the *reciprocal* of the second fraction.

Example 4:

$$\frac{3p}{10p} \div \frac{6p}{5} = \frac{3p}{10q} \times \frac{5}{6p} = \frac{15p}{60pq} = \frac{1}{4q}$$

1 Add or subtract these fractions. Simplify your answer if possible.

a $\frac{x}{5} + \frac{2x}{5}$ _____

b $\frac{x}{10} + \frac{7x}{10}$ _____

c $\frac{5x}{12} - \frac{x}{12}$ _____

d $\frac{17x}{20} - \frac{9x}{20}$ _____

e $\frac{x}{3} + \frac{4x}{3} + \frac{7x}{3}$ _____

2 Write each of these expressions as a single fraction.

a $\frac{x}{5} + \frac{x}{2}$ _____

b $\frac{2x}{3} + \frac{x}{5}$ _____

c $\frac{2x}{3} + \frac{5x}{6}$ _____

d $\frac{5p}{6} - \frac{3p}{8}$ _____

e $\frac{7x}{8} - \frac{17x}{24}$ _____

3 Multiply these fractions and write the answer in its simplest possible form.

a $\frac{x}{3} \times \frac{x}{2}$ _____

b $\frac{x}{6} \times \frac{3}{4}$ _____

c $\frac{2}{5} \times \frac{x}{4}$ _____

d $\frac{5}{x} \times \frac{x}{2}$ _____

e $\frac{6x}{5y} \times \frac{2x}{3y}$ _____

f $\frac{3x}{5} \times \frac{10}{9x}$ _____

g $\frac{5x}{2y} \times \frac{4y}{25}$ _____

4 Divide these fractions and write the answers in their simplest possible form.

a $\frac{x}{8} \div \frac{x}{6}$ _____

b $\frac{5x}{2} \div \frac{x}{4}$ _____

c $\frac{2x}{3y} \div \frac{4x}{5y}$ _____

d $\frac{2pq}{r} \div \frac{4qr}{5p}$ _____