Percentages 1

Student Name	Class	Score
Parent Signature	Date	

4:01 | Review of Percentages

Outcome NS 4.3

The word 'percent' means out of 100. A percentage is a 'part out of every hundred'.
   The symbol for percent is %.

Example 1: 50% means 50 out of 100, or $\frac{50}{100}$. We know that $\frac{50}{100}$ simplifies to $\frac{1}{2}$, so we use the terms 50% and one-half interchangeably.

Example 2: Change 45% to a fraction. Simplify if possible.

$$45\% = \frac{45}{100}$$

$$\frac{45}{100} = \frac{9}{20}$$

(Simplify by noticing that 5 goes into both 45 and 100.)

To change a fraction to a percentage, multiply it by 100 (or $\frac{100}{1}$).

Example 3: Change $\frac{3}{4}$ to a percentage.

$$\frac{3}{4} \times \frac{100}{1} = \frac{300}{4}$$

$$= 75\%$$

To change a percentage to a decimal, write it out of 100, and do the division.

Example 4: Change 86% to a decimal.

$$\frac{86}{100} = 86 \div 100 = 0.86$$

Remember that when dividing a number by 100, the decimal point will move two places to the left.
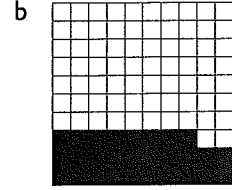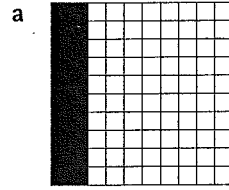   To change a decimal to a percentage the rule is 'multiply by 100'.

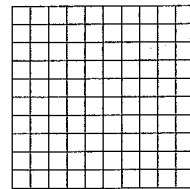**Example 5:** Change 0.43 to a percentage.
$$0.43 \times 100 = 43\%$$

Remember that when multiplying a decimal by 100, the decimal point will move two places to the right.

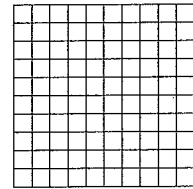**1** What percentage of the whole square is shaded in each of these diagrams?

a   b

_____

2 Shade these grids to show these percentages.

a 75%   b 6%

3 Write these percentages in order from smallest to largest.

a 53%, 7%, 49%, 17%, 70%

_____

b 12%, $6\frac{1}{2}$, $33\frac{1}{3}$, 9%

_____

4 Forty-five percent of the students in the school canteen are girls. What percentage are boys?

5 Of the journeys of people from Melbourne to Sydney last year, 43% were by car, 7% by bus, and 18% by train. What percentage of journeys were by air?

6 Write these percentages as fractions. Simplify if possible.

a 17% _____   b 30% _____

   _____   _____

c 99% _____   d 49% _____

   _____   _____

e 75% _____   f 25% _____

   _____   _____

g 1% _____   h 5% _____

   _____   _____

i 6% _____   j 60% _____

   _____   _____

7 Seventy-six percent of drivers at a toll plaza have the correct change.

a What fraction have the correct change?

_____

b What fraction do not have the correct change?

_____

8 Change these fractions to percentages.

a $\frac{2}{5}$ _____   b $\frac{3}{10}$ _____

c $\frac{49}{100}$ _____   d $\frac{3}{25}$ _____

e $\frac{1}{4}$ _____   f $\frac{7}{20}$ _____

9 Change these percentages to decimals.

a 50% _____   b 29% _____

c 4% _____   d 8% _____

10 Change these decimals to percentages.

a 0.53 _____   b 0.3 _____

c 0.01 _____   d 0.7 _____

Fun Spot

In the year 2004 Caterina and her much older brother Edward will each become as old as the sum of the digits of their own year of birth.
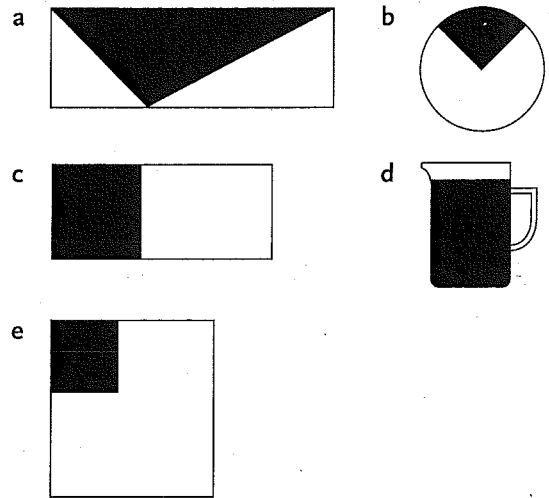
How old will each one be, and in what years were they born?

4:02 | Estimating Percentages

Outcome NS 4.3

**1** Match the shaded part in each of these diagrams with the most likely percentage from the box.

a   b

c   d

e

| 15% 25% 40% 50% 85% |

2 This table gives information about the approximate populations of the states/territories of Australia (based on 2001 census).

	Population	Percentage
ACT	312 000	
NSW	6 372 000	
NT	211 000	
QLD	3 655 000	
SA	1 467 000	
TAS	457 000	
VIC	4 645 000	
WA	1 851 000	
Total		

The information has been used to produce the pie graph below. Use the graph to estimate the percentage of Australia's population that lives in each state/territory. Write your answers in the table.
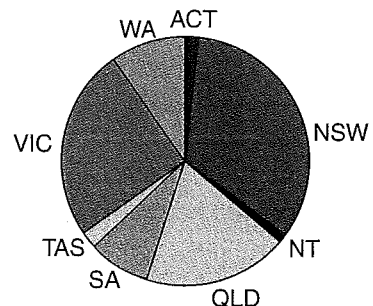
Percentages 2

Student Name		Class	Score
Parent Signature		Date	

4:03 | Harder Conversions

Outcome NS 4.3

Example 1: Change $33\frac{1}{3}\%$ to a fraction.

Answer:

$$33\frac{1}{3} = \frac{100}{3}$$

$$33\frac{1}{3}\% = \frac{100}{3} \div 100$$

$$= \frac{100}{3} \times \frac{1}{100}$$

$$= \frac{1}{3}$$

Use a calculator for problems where fractions do not change exactly to whole number percentages.

Example 2: Change $\frac{2}{7}$ to a percentage. Round the answer to two decimal places.

Answer: The working is:

$$\frac{2}{7} \times \frac{100}{1} = \frac{200}{7}$$

On a calculator the keys to use are:

[2] [÷] [7] [×] [1] [0] [0] [=]

The result is 28.57142857

$$\frac{2}{7} = 28.57\% \text{ (2 dec. pl.)}$$

1 Change these percentages to decimals.

a 125% _____ b 69.2% _____

c 4.89% _____ d 117.2% _____

2 Change these decimals to percentages.

a 0.649 _____ b 1.56 _____

c 0.0034 _____ d 0.071 _____

3 Write these percentages as fractions in their simplest forms.

a $12\frac{1}{2}\%$ _____ b $16\frac{2}{3}\%$ _____

c $2\frac{1}{4}\%$ _____ d $10\frac{1}{5}\%$ _____

4 Change these fractions to percentages. Round each answer to two decimal places.

a $\frac{1}{3}$ _____ b $\frac{4}{9}$ _____

c $\frac{3}{7}$ _____ d $\frac{5}{6}$ _____

5 This table shows Ailsa's results in four maths tests this year.

Topic	Mark out of total	Percentage
Probability	$\frac{11}{20}$	
Angles	$\frac{24}{30}$	
Area	$\frac{27}{50}$	
Fractions	$\frac{12}{25}$	

a Complete the table to show each mark as a percentage.

b In which topic was Ailsa's lowest mark (as a percentage)?

_____

c Which topic do you think Ailsa understood best?

_____

d Explain what you need to assume to answer part c.

_____

_____

_____

_____

4:04 | Finding a Percentage of a Quantity

Outcome NS 4.3

To work out a percentage of a quantity you multiply the percentage by the quantity.

Example: Calculate 30% of $120.

Answer:

$$30\% \times 120 = \frac{30}{100} \times \frac{120}{1}$$

$$= \frac{3600}{100}$$

$$= \$36$$

Note: this working uses fractions—you could also use decimals and multiply by 0.3.

Alternatively you can use a calculator.

[3][0][÷][1][0][0][×][1][2][0][=]

percentage   quantity

Note: the word 'of', when used with fractions and percentages, means you multiply.

Work out the following.

1 70% of 50 _____

2 32% of 600 _____

3 24% of 300 _____

4 5% of 200 _____

5 42% of $12 _____

6 60% of $18 _____

7 7% of 33 m _____

4:05 | Applications of Percentages

Outcomes NS 4.3, WMS 4.2

1 Thirty-nine percent of the population have type O blood and 2% have type AB blood. Write these percentages as decimals.

Type O: _____   Type AB: _____

2 The fuel tank on a motor-mower holds exactly 1 litre. 19.7% of this is high-grade oil.

a Write this percentage as a decimal. _____

b How much oil, in litres, is in the engine? ____

3 A sheep farmer finds that 1.2 lambs are born on his farm for every 1 ewe. Use this information to write down the 'lambing percentage' on the farm.

4 The distance from Sydney to Adelaide by the Hume and Sturt Highways is 1415 km; 1137 km of the journey is within NSW.

What percentage (to the nearest whole number) is within NSW?

_____

_____

5 A soccer player scores with 40% of his shots on goal. How many times would you expect him to score if he shoots for goal 30 times altogether?

6 A church suggests its members pay a 'tithe' of 10% of their income. How much would a member with an income of $35 000 be expected to pay?

_____

7 A serious drought affected avocado production so that the total weight of fruit at an orchard was only 70% of last year's figure—6500 kg. Calculate the total weight produced this year.

_____

_____

8 Instructions for mixing concrete say: The amount of cement added to the mixture should be 30% of the weight of gravel. How much cement should be added to 150 kg of gravel?

_____

_____

9 Membership fees for senior citizens at a social club are set at 80% of the level charged to ordinary members. If the fee for ordinary members is $90, how much are senior citizens expected to pay?

_____

10 Sixty-seven percent of the 450 members of a sports club have paid their subscriptions. How many more members would need to pay to raise the percentage who have paid to 75%?

_____

Percentages 3

Student Name	Class	Score
Parent Signature	Date	

4:06 | Percentage Composition

Outcome NS 4.3

> To write one quantity as a percentage of another:
> • write it as a fraction first
> • change the fraction into a percentage
>
> **Example:** Eight apples out of a bag of 25 are rotten. What percentage are rotten?
>
> $$\frac{8}{25} \times \frac{100}{1} = 32\%$$

1 What percentage of $70 is $14? _____

2 Express the first quantity as a percentage of the second.

  a 6 kg out of 40 kg _____

  b 30 seconds out of 125 seconds _____

  c 2.4 ha out of 6 ha _____

  d $294 out of $840 _____

3 What percentage is 30 cm out of 12 m? _____

4 A retailer sold a washing machine for $800 and received a commission of $40. What percentage commission is this? _____

5 Wayne was cycling the 75-km distance from Maitland to Nelson Bay. He got blisters after 45 km and had to stop. What percentage of the distance had he completed? _____

6 A six-month old baby spends 18 hours a day sleeping. What percentage of the day is this?

_____

7 Marcel tests a coin by tossing it 80 times to see if it is 'fair'. If it is 'fair' he would expect it to land with heads facing up about half the time.

  a The coin lands with heads facing up 32 times. Calculate the percentage of times the coin lands with heads facing up. _____

  b What should the percentage be, approximately, for a fair coin? _____

8 The school groundsman is marking out the 300-m perimeter of a soccer field. He runs out of paint after 180 m.

  a What percentage has been painted?

  b What percentage remains to be painted?

4:07 | Percentage Change

Outcome NS 4.3

> Percentages are often used to express increases and decreases.
>   For example, 'The number of sit-ups I can do at one time has increased by 10% this year'; 'The price of the skateboard I want to buy has decreased by 25%'.
>
> **Example 1:** Increase 450 cm by 18%.
> **Answer:**
>
> $$18\% \text{ of } 450 \text{ is } \frac{18}{100} \times 450 = 81$$
>
> Add this to the original amount (450 cm):
> $$450 + 81 = 531 \text{ cm}$$
>
> Alternatively, to increase by 18% you end up with 118% of the original quantity, so you could get the answer directly by multiplying by 1.18 (which is 118% written as a decimal).
>
> **Example 2:** The area of a marine reserve is reduced by 20%. It used to be 14 000 ha. What is its area now?
> **Answer:**
> The decrease is 20% of 14 000 =
> $$\frac{20}{100} \times 14\ 000 = 2800$$
> The area now = 14 000 − 2800
> $$= 11\ 200 \text{ ha}.$$
>
> Alternatively, to decrease by 20% you end up with (100 − 20)% = 80% of the original quantity, so you could get the answer directly by multiplying by 0.80 (which is 80% written as a decimal).

1 Calculate the increased amounts.

a Increase $40 by 15%.

_____

b Increase 6500 L by 25%.

_____

c Increase 450 m^2 by 100%.

_____

d Increase 512 kg by 4%.

_____

2 Calculate the decreased amounts.

a Decrease $500 by 10%.

_____

b Decrease 88 L by 75%.

_____

c Decrease 6400 ha by 3%.

_____

d Decrease 73 000 kg by 16%.

_____

3 A supermarket worker on an hourly rate of $13.50 was given a pay increase of 8%. Calculate the new hourly rate.

_____

4 The owners of a paper mill with 580 employees decide to reduce employee numbers by 15%.

a How many employees will lose their jobs?

_____

b How many employees will the paper mill now have?

_____

5 Telstra offer a 1% discount to customers who pay by direct debit. The Williams family have a bill of $73.92.

a How much will the Williams family save by paying by direct debit?

_____

b How much will they pay Telstra?

_____

6 A manufacturer sells plastic bottles of mouthwash. Each bottle contains 350 mL. It decides to increase the amount of mouthwash in a bottle by 10%. Complete this sentence on the label.

Mint Breath Mouthwash
10% extra
Now contains ...... mL

7 Footsteps Ltd is a shoe retailer. Once a year they hold a special sale in which all prices are reduced by 25%. Calculate the new price for each of these items.

a cross-trainers: $129

_____

b boat shoes: $69

_____

c six-pack of cotton socks: $23

_____

8 A retired person has a pension which is adjusted once a year to keep up-to-date with inflation. At the start of a year it is $468 a week. If inflation during the year is 6%, what amount should the pension be increased to?

_____

9 A fence which is 1.5 m high is raised to a new height of 1.8 m.

a Calculate the increase in height.

_____

b What is the percentage increase in height?

_____

10 The roll at Menzies High School has increased from 1200 students to 1500 students. Calculate the percentage increase.

_____

11

Calculate the percentage price reduction on this TV set.

_____

Percentages 4

Student Name		Class	Score
Parent Signature		Date	

4:08 | Further Applications of Percentages

Outcomes NS 4.3, WMS 4.1–4.5

Percentages can be used to create a misleading impression.

You should always take the original amount into consideration.

Example 1: A petrol station offers customers a choice between a $2 'cash off' discount or 5% off.

A customer who buys $50 of petrol should choose the 5% discount, because the saving would be 5% of $50 = $2.50.

A customer who buys $30 of petrol should choose the $2 'cash off' because this is a greater saving than 5% of $30 = $1.50.

Profit and Loss

In business, these special terms are used:
- **Original cost** = how much a business pays for an item
- **Selling price** = how much the business sells the item for
- The difference between these is the **profit**.
- Profit = selling price − original cost

Overall, the profit or the loss is the difference between the income of a business and its expenditure.

When the profit is expressed as a percentage it is always of the original cost.

Example 2: A tyre dealer buys tyres for $37 and sells them for $49. Calculate the percentage profit.

Answer: Profit = selling price − original cost
$$= 49 - 37$$
$$= \$12$$

$$\text{Profit\%} = \frac{\text{profit}}{\text{original cost}} \times 100$$
$$= \frac{12}{49} \times \frac{100}{1}$$
$$= 24.5\% \ (1 \text{ dec. pl.})$$

If the selling price is less than the original cost price the business has made a loss.

Example 3: A phone-card collector bought some cards for $240. A year later they were sold at a loss of 30%. What price did they sell for?

Answer: Loss = 30% of $240 = $\frac{30}{100} \times 240 = \$72$

Selling price = original cost − loss = 240 − 72 = $168

1 Explain what is wrong or misleading about these advertisements.

a

Special deal on Chocolate Raisins

20% more in packet

b

Arancia orange juice

98% pure

2

Headline: '60% of people opposed to new motorway'

This headline was based on a survey of five people only. How would the headline change if one of the five people changed their mind?

3

Headline: 'Dramatic decrease in speeding'

In July one day, a speed camera takes photos of 24 speeding cars out of 80 passing by. In August one day, it takes photos of eight speeding cars out of 25. Has the rate of speeding increased or decreased from July to August? Explain using percentages.

4 Calculate the profit or loss as a percentage of the original cost.

a   original cost = $50, selling price = $62.50

_____

b   selling price = $567, original cost = $819

_____

5 A whiteware dealer makes a profit of 15%. Calculate the selling price of a clothes dryer with an original cost of $580.

6 Discount Disasters sell goods at a loss of 18% on behalf of manufacturers with cancelled orders. Calculate the selling price for bedspreads with an original cost of $375.

Fun Spot

Australian and New Zealand 20c coins are the same size and weight. There are 12 of each in a bag. What is the smallest number you would need to take out of the bag, without looking, to make sure you would have at least four Australian coins?

4:09A | Commission

Some workers who sell products like cars and houses, or services like insurance, are paid a percentage of the amount sold.

Example: A travel agent is paid 2.5% commission on all travel she sells. How much is the agent paid for sales of $18 000?

Answer:

$2.5\% \text{ of } \$18\,000 = \dfrac{2.5}{100} \times 18\,000 = \$450$

1 Calculate the commission payable to these workers.

a   a car salesman who earns 7% on sales of $60 000

_____

b   a sales rep who earns 15% on sales of $12 500

_____

c   a bookseller who gets a 35% commission on sales of textbooks worth $670 000

_____

2 A real estate firm charges a flat fee of $500 per sale, plus $2\frac{1}{2}$ % commission on the selling price. Calculate the amount the firm earns on a house that it sells for $160 000.

_____

3 Fast-track Couriers Ltd pay their drivers 45 cents for every kilometre travelled, plus 55% of the delivery fees charged. Calculate the earnings for a driver who travels 234 km and makes deliveries that have been charged a total of $160.

_____

_____

_____

4 A medical insurance policy will reimburse clients 80% of fees paid for medical treatment and 50% of fees paid for dental treatment. Calculate the refund due to a client who has paid $1200 for medical treatment and $365 at the dentist.

_____

_____

_____

5 A used-car salesman is paid a 5% commission on each car he sells. If he is paid $750 on a sale, calculate the amount the car sold for.

_____

_____

_____

Fun Spot

In a recent pet show, 30% of the 200 pets entered were cats. Twenty pets were chosen as finalists, and 40% of these were cats. Work out the percentage of cats that became finalists.

Percentages 5

Student Name		Class	Score
Parent Signature		Date	

4:09B | Simple Interest

Outcome NS 4.3

> Money deposited at a bank earns interest. Alternatively, if you borrow money you often have to pay interest.
>
> Simple interest involves making equal payments for equal time periods. We use percentages to express the *rate* of interest.
>
> Simple interest = interest for 1 year × number of years
>
> **Example:** Rhonda makes a deposit of $450 at the bank. The interest rate is 6%. The money will be left in the bank for 5 years. Each year the bank posts Rhonda a cheque for the interest. How much interest will Rhonda earn?
>
> **Answer:**
> $$\text{Interest for 1 year} = 6\% \text{ of } 450$$
> $$= 0.06 \times 450$$
> $$= \$27$$
> $$\text{Interest for 5 years} = 5 \times \$27$$
> $$= \$135$$

1 Calculate the simple interest for each of these.

a $3000 invested at 12% for 2 years

b $50 000 borrowed at 6% for 4 years

c $850 invested at 7.5% for 2 years

d $200 000 borrowed at 9.5% for 12 years

2 Joan makes a deposit of $500 at her bank. The interest rate is 6%, and she agrees to leave the money with the bank for 2 years. How much simple interest will she earn?

3 Tony borrows $4000 from his father to go on a school trip with the rugby team to France. The arrangement is that he will have to pay the money back in 10 years' time (when he has a job) and the interest rate will be 6% over the period of the loan.

a How much interest will be owing when Tony repays the loan?

b How much will Tony have to repay altogether?

4 The Neurological Society are left $120 000 from a dead person's estate. They put it in the bank for $1\frac{1}{2}$ years at an interest rate of 5% per annum while they decide what to do with it. How much money will they have *altogether* when the $1\frac{1}{2}$ years is finished?

5 This table shows details of four different loans. Calculate the values marked a, b, c and d.

	Interest	Principal	Rate	Time period
Loan 1	a	$1 500	6%	3 years
Loan 2	b	$3 840	12.5%	5 years
Loan 3	$5040	$63 000	c	1 year
Loan 4	$1400	$7 000	d	2.5 years

$a =$

$b =$

$c =$

$d =$

4:10 | Finding a Quantity If a Percentage Is Known

Outcomes NS 4.3, WMS 4.2

First find 1% (this involves dividing). Then multiply by 100 to find 100% (the whole quantity).

Example: 32% of the crowd at a rugby game were season-ticket holders. There were 5760 season-ticket holders at the game. Calculate the size of the crowd.

Answer:

$$32\% \text{ of the crowd} = 5760$$
$$1\% \text{ of the crowd} = 5760 \div 32$$
$$= 180$$
$$\text{The whole crowd} = 100 \times 180$$
$$= 18\ 000$$

1 Determine the value of x (the original quantity).

a 4% of x is $24 _____

_____

b 25% of x is 200 m _____

_____

c 41% of $x = 1927$ _____

_____

d 6% of $x = 900$ _____

_____

2 About 28% of international phone calls from Australia are to North America. If there were 47 890 calls from Australia to North America last week, estimate the total number of international calls from Australia to the nearest thousand.

3 A real estate agent is paid 3.4% commission when selling a house. On one particular house sale the agent is paid $5984. Calculate the price that the house sold for.

4 A fast-food restaurant estimates that 45% of its customers buy soft drinks with their purchases. Last week 6200 customers did not buy soft drinks. How many customers did they have?

5 Janet and two friends are moving into a flat together and looking for new and second-hand furniture. Janet agrees to pay 30% of the cost of the dining room furniture, 25% of the cost of the TV and 65% of the cost of the washing machine. This shows how much Janet pays for each item:

Dining room furniture:	$225
TV set:	$99.75
Washing machine:	$390

Calculate the full cost of each item.

6 A sports doctor estimates that 35% of rugby players get injuries which require hospital treatment each season. In a survey of players, 260 *did not* have this type of injury. How many players were surveyed?

Fun Spot—The Rule of 72

The Rule of 72 has been used by banks and financial advisers for a long time. It gives a good indication of how many years it will take to double your money at any given interest rate.

To work out the number of years it takes for money to double, divide 72 by the interest rate.

If the interest rate is 6%, it will take you about 12 years to double your money.

If the interest rate is only 4%, it will take about 18 years.

1 How long will it take you to double your money if the interest rate is 9%?

_____

2 How long if the interest rate is 12%?

_____

3 Write the Rule of 72 as a formula. Use y for the number of years, and r for the interest rate.

_____