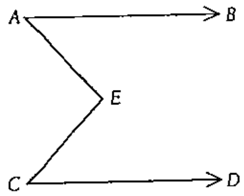
Foundation Worksheet

5:04 | Non-numerical Proofs

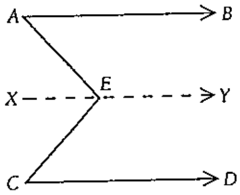
Name: ______________________________ Class: ____________

Example

Prove $\angle AEC = \angle BAE + \angle DCE$.



Proof Draw a third parallel line, XY, through E.

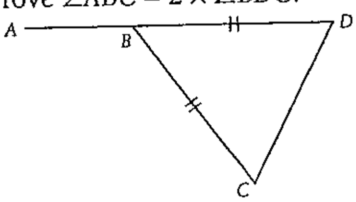


Now $\angle BAE = \angle AEX$
(alternate $\angle$s, $AB \parallel XY$)
and $\angle DCE = \angle CEX$
(alternate $\angle$s, $CD \parallel XY$)

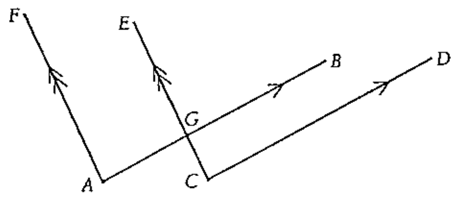
Then $\angle AEC = \angle AEX + \angle CEX$
$= \angle BAE + \angle DCE$

Exercise

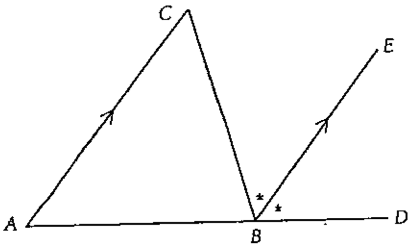
1 Prove $\angle ABC = 2 \times \angle BDC$.



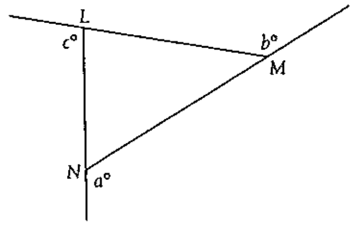
2 Prove $\angle FAB = \angle ECD$.



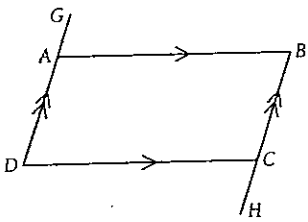
3 Prove $\angle CAB = \angle ACB$.



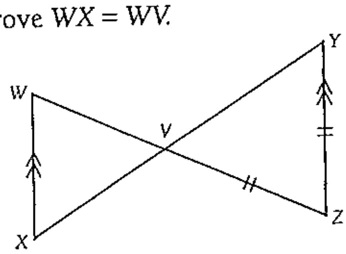
4 Prove $a + b + c = 360$.



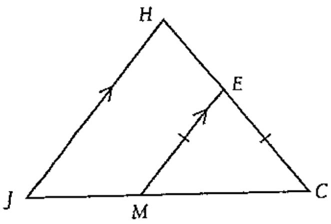
5 Prove $\angle BAG = \angle HCD$.



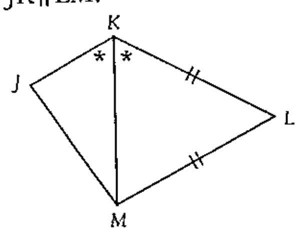
6 Prove $WX = WV$.



7 Prove $HJ = HC$.



8 Prove $JK \parallel LM$.



$\angle ABC = \angle BCD + \angle BDC$
(exterior angle of $\triangle BCD$)
but $\angle BCD = \angle BDC$
(isosceles triangle)
$\therefore \angle ABC = \angle BDC + \angle BDC$
$= 2 \times \angle BDC$

$\angle ACB = \angle CBE = *$
(alternate $\angle$s, $AC \| BE$)
and $\angle EBD = \angle CAB = *$
(corresponding $\angle$s, $AC \| BE$)
$\therefore \angle CAB = \angle ACB$ (= *)

$\angle BAD = 180° - \angle BAG$ (straight angle)
and $\angle BCD = 180° - \angle HCD$ (straight angle)
but $\angle BAD = \angle BCD$
(opposite $\angle$s of parallelogram)
$\therefore 180° - \angle BAG = 180° - \angle HCD$
$\therefore \angle BAG = \angle HCD$

$\angle HJC = \angle EMC$
(corresponding $\angle$s, $JH \| ME$)
$\angle ECM = \angle EMC$
(base $\angle$s of isosceles triangle)
$\therefore \angle HJC = \angle ECM$
$\therefore \triangle HJC$ is isosceles
$\therefore HJ = HC$

2 $\angle FAB = \angle EGB$
(corresponding $\angle$s, $FA \| EC$)
and $\angle EGB = \angle ECD$
(corresponding $\angle$s, $AB \| CD$)
$\therefore \angle FAB = \angle ECD$ (= $\angle EGB$)

4 $\angle LMN = 180° - b°$ (straight angle)
$\angle MNL = 180° - a°$ (straight angle)
$\angle NLM = 180° - c°$ (straight angle)
Then angle sum of $\triangle LMN$ gives
$180 - b + 180 - a + 180 - c = 180$
$540 - a - b - c = 180$
$540 - 180 = a + b + c$
ie $a + b + c = 360$

6 $\angle ZYV = \angle WXV$
(alternate $\angle$s, $WX \| YZ$)
and $\angle YVZ = \angle WVX$ (vertically opposite)
but $\angle ZYV = \angle YVZ$
(base $\angle$s of isosceles triangle)
$\therefore \angle WXV = \angle WVX$
$\therefore \triangle WVX$ is isosceles
$\therefore WX = WV$

8 $\angle LKM = \angle LMK = *$
(base $\angle$s of isosceles triangle)
$\therefore \angle JKM = \angle LMK$ (= *)
but these are alternate angles
$\therefore JK \| LM$