

SYDNEY GRAMMAR SCHOOL



2013 Half-Yearly Examination

Yr 8

FORM II
MATHEMATICS

Tuesday 21st May 2013

General Instructions

- Writing time — 1 hour 30 minutes
- Write using black or blue pen.
- Calculators are not to be used.

Total — 96 Marks

- All questions may be attempted.
- All necessary working should be shown.
- Start each question on a new page.

Collection

- Write your name, class and master on each page of your answers.
- Staple your answers in a single bundle.
- Write your name and master on this question paper and submit it with your answers.

| | | | |
|---------|---------|---------|--------|
| 2A: BR | 2B: REJ | 2C: LYL | 2D: SG |
| 2E: GMC | 2F: PKH | 2G: KWM | 2H: SO |
| 2I: LRP | 2J: DNW | | |

Checklist

- Writing paper required.
- Candidature — 194 boys

Examiner
GMC

QUESTION ONE (12 marks) Start a new page.

(a) Simplify:

(i) $4 + 5 \times 3$

(ii) $3 - (-5)$

(iii) $6 \div (-3)$

(b) (i) Express $1\frac{2}{3}$ as an improper fraction.

(ii) Express 8% as a decimal.

(c) Simplify:

(i) $3a + 5a$

(ii) $2x - x$

(iii) $(3a)^2$

(iv) $12x \div 2x$

(d) Evaluate:

(i) $1.2 \div 3$

(ii) $\frac{6}{7} - \frac{4}{7}$

(iii) 10% of 25

QUESTION TWO (12 marks) Start a new page.

(a) Express 15% as a fraction in lowest terms.

(b) Simplify:

$$(i) \ 3x - 2y - x + y$$

$$(ii) \ \frac{5x}{6} - \frac{2x}{3}$$

(c) Expand:

$$(i) \ 4(x + 7)$$

$$(ii) \ 3x(x - 2)$$

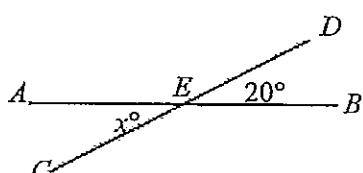
(d) Evaluate the following when $a = 3$ and $b = 4$:

$$(i) \ \frac{ab}{2}$$

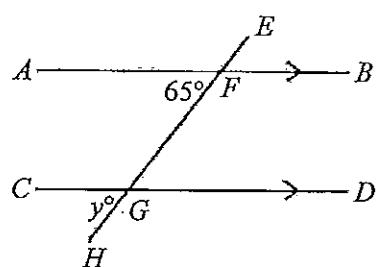
$$(ii) \ b - 2a$$

(e) Find the value of the pronumeral in each diagram below. Give reasons for your answers.

(i)



(ii)

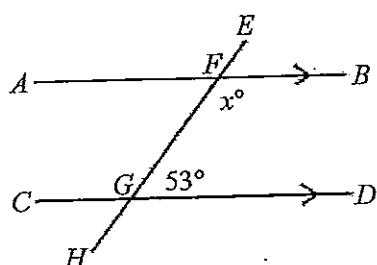


QUESTION THREE (12 marks) Start a new page.

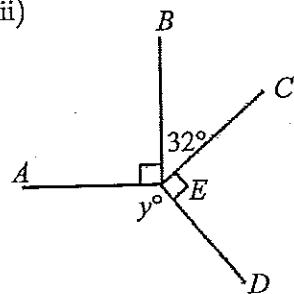
- (a) Find 80% of \$80.
- (b) Solve:
 - (i) $3x - 5 = 4$
 - (ii) $18x - 5 = 4x + 2$
- (c) Evaluate $1\frac{1}{2} + 2\frac{2}{3}$. Express your answer as a mixed numeral.

- (d) Find the value of the pronumeral in each diagram below. Give reasons for your answers.

(i)



(ii)



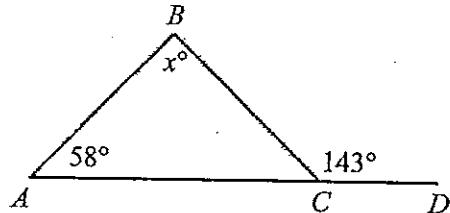
QUESTION FOUR (12 marks) Start a new page.

(a) Factorise fully:

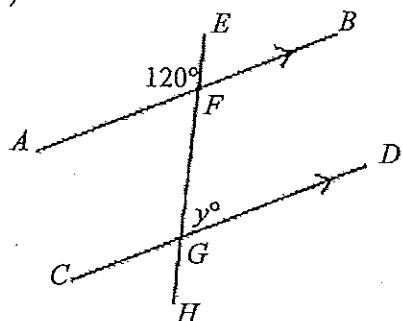
- (i) $-2x - 8$
- (ii) $9abc + 6b$
- (iii) $4p^3 - p^2$

(b) Find the value of the pronumeral in each diagram below. Give reasons for your answers.

(i)



(ii)



(c) (i) Solve $2x + 1 \geq -3$.

(ii) Graph your solution to part (i) on a number line.

(d) Decrease \$90 by 25%.

QUESTION FIVE (12 marks) Start a new page.

- (a) Write an expression for the sum of a and the product of b and c .
- (b) Find 320% of 5.
- (c) Andrew deposited \$1000 into a bank account earning 7.5% simple interest per annum.
How much interest did he earn over 5 years?
- (d) Fully simplify the following:
- (i) $\frac{3x^2y}{4} \times \frac{2x}{y}$
- (ii) $\frac{x}{6} \div \frac{2x}{3}$
- (e) Three times the sum of a certain number and four is twenty-seven. Let the number be x .
- (i) Write an equation using the information provided.
- (ii) Solve the equation to find the number.

QUESTION SIX (12 marks) Start a new page.

(a) Find:

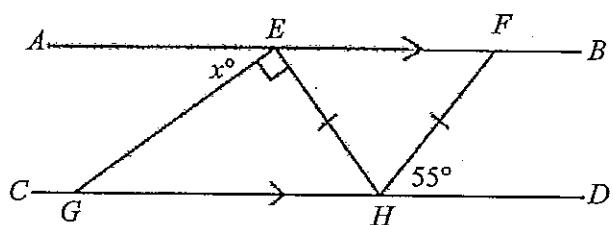
(i) $0.2 \div 0.05$

(ii) $1\frac{2}{3} - \frac{2}{3} \div \frac{4}{9}$

(b) Solve $2(x + 3) - 3(x - 1) = 2$.

(c) Solve $x - 2 < 2x$.

(d) Find the value of x in the diagram below. Give reasons for your answers.



(e) Place $\frac{3}{5}$, $\frac{4}{7}$ and $\frac{7}{12}$ in order from lowest to highest.

QUESTION SEVEN (12 marks) Start a new page.

(a) Solve $4x - 3(2 - x) = 8 - 5(x - 2)$.

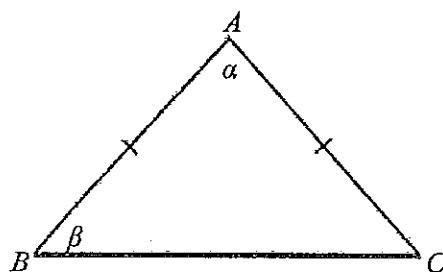
- (b) A bag contains 75 green marbles and 125 blue marbles. A certain quantity of blue marbles is removed from the bag so that 40% of the remaining marbles in the bag are blue.

Let x be the number of blue marbles removed from the bag.

(i) Write an equation for the number of blue marbles left in the bag.

(ii) Hence solve your equation in part (i) to find x .

- (c) In the triangle below $AB = AC$, $\angle BAC = \alpha$ and $\angle ABC = \beta$.



(i) Find α in terms of β showing all reasons.

(ii) Find the values of β that make α obtuse. Express your answer as an inequality.

- (d) The harmonic mean H of two numbers a and b is calculated using the formula below.

$$\frac{2}{H} = \frac{1}{a} + \frac{1}{b}$$

Find H when $a = 2$ and $b = 3$.

QUESTION EIGHT (12 marks) Start a new page.

- (a) A triangle is to be drawn with the largest angle twice the size of the smallest and with the third angle the average of the other two. By using an equation, find the size of each angle.
- (b) The conversion from degrees Celsius to degrees Fahrenheit is given by the formula

$$F = \frac{9}{5}C + 32,$$

where C is the temperature in degrees Celsius and F is the temperature in degrees Fahrenheit.

Showing all your working, find which temperature is represented by the same number in both Celsius and Fahrenheit.

- (c) An ice sculpture loses 20% of its mass each day. If the sculpture weighs 3.6 tonnes at the end of two days, calculate its original mass.
- (d) A family consists of one adult and four children each of whom has reached their second birthday. If the product of their ages in years is 111 111, find the difference in age between the oldest child and the youngest child.

END OF EXAMINATION

Form II Half Yearly Solutions . 2013

$\boxed{1}$ a) i) $4 + 5 \times 3 = 19$ ✓
 ii) $3 - (-5) = 8$ ✓
 iii) $6 \div (-3) = -2$ ✓

b) i) $1\frac{2}{3} = \frac{5}{3}$ ✓
 ii) $8\% = 0.08$ ✓

c) i) $3a + 5a = 8a$ ✓
 ii) $2x - x = x$ ✓
 iii) $(3a)^2 = 9a^2$ ✓
 iv) $12x \div 2x = 6$ ✓

d) i) $1.2 \div 3 = 0.4$ ✓
 ii) $\frac{6}{7} - \frac{4}{7} = \frac{2}{7}$ ✓
 iii) 10% of 25 = 2.5. ✓

12

$\boxed{2}$ a) $15\% = \frac{15}{100}$ ✓
 $= \frac{3}{20}$ ✓

b) i) $3x - 2y - x + y = 2x - y$ ✓
 ii) $\frac{5x}{6} - \frac{3x}{3} = \frac{5x}{6} - \frac{4x}{6}$
 $= \frac{x}{6}$ ✓

c) i) $4(x+7) = 4x + 28$ ✓
 ii) $3x(x-2) = 3x^2 - 6x$ ✓

d) i) $\frac{3 \times 4}{2} = 6$ ✓
 ii) $4 - 2 \times 3 = -2$ ✓

e) i) $x = 20$ (vertically opposite angles) ✓✓
 ii) $y = 65$ (corresponding angles in parallel lines) ✓
 OR corresponding angles $AB \parallel CD$

12

② a) $80\% \text{ of } \$80 = \frac{8}{10} \times 80 \checkmark$
 $= \$64 \checkmark$

b) i) $3x - 5 = 4$
 $3x = 9 \checkmark$
 $x = 3 \checkmark$

ii) $13x - 5 = 4x + 2$
 $9x = 7 \checkmark$
 $x = \frac{7}{9} \checkmark$

c) $1\frac{1}{2} + 2\frac{2}{3} = 3 + \frac{1}{2} + \frac{2}{3}$
 $= 3 + \frac{3}{6} + \frac{4}{6}$
 $= 3 + 1\frac{1}{2}$
 $= 4\frac{1}{2} \checkmark \checkmark$

d) i) $x^\circ + 53^\circ = 180^\circ$ (co-interior angles, AB || CD) \checkmark
 $x = 127 \checkmark$

ii) $y^\circ + 90^\circ + 90^\circ + 32^\circ = 360^\circ$ (adjacent angles in a revolution) \checkmark
or revolution
 $y = 148 \checkmark$

12

4) a) i) $-2x - 8 = -2(x + 4)$ ✓

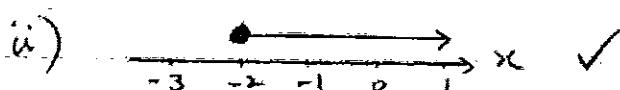
ii) $9abc + 6b = 3b(3ac + 2)$ ✓

iii) $4p^3 - p^2 = p^2(4p - 1)$ ✓

b) i) $x^\circ - 58^\circ = 143^\circ$ (external angle is sum of
opposite internal angles) ✓
 $x = 85$ ✓

ii) $\angle BFG = 120^\circ$ (vertically opposite angles) ✓
 $y + 120^\circ = 180^\circ$ (co-interior angles, AB || CD)
 $y = 60$ ✓

c) i) $2x + 1 \geq -3$
 $2x \geq -4$ ✓
 $x \geq -2$ ✓



d) $\$90 \times 75\% = \$90 \times \frac{3}{4}$ ✓
 $= \$67.50$ ✓

12

15) a) $a + bc$ ✓

b) $\frac{24}{60} = \frac{4}{10}$ ✓
 $= 40\%$ ✓

So Jack spent 40% of his time studying French.

c) $1000 \times 0.075 \times 5 = 375$ ✓✓
So Andrew earned \$375 interest

d) i) $\frac{3x^2y}{4} \times \frac{2x}{y} = \frac{3x^3}{2}$ ✓✓

ii) $\frac{x}{6} \div \frac{2x}{3} = \frac{x}{6} \times \frac{3}{2x}$ ✓
 $= \frac{1}{4}$ ✓

e) i) $3(x+4) = 27$ ✓

ii) $x+4 = 9$ ✓
 $x = 5$

So the number is 5

12

6) a) i) $0.2 \div 0.05 = 4$ ✓✓
 ii) $1\frac{2}{3} \div \frac{2}{3} \div \frac{4}{9} = 1\frac{2}{3} - \frac{2}{3} \times \frac{9}{4}$
 $= 1\frac{2}{3} - \frac{3}{2}$ ✓
 $= 1\frac{2}{3} - 1\frac{1}{2}$
 $= \frac{1}{6}$ ✓

b) $2(x+3) - 3(x-1) = 2$
 $2x + 6 - 3x + 3 = 2$ ✓
 $-x + 9 = 2$
 $-x = -7$
 $x = 7$ ✓

c) $x-2 < 2x$ $x-2 < 2x$
 $-x-2 < 0$ OR $-2 < x$
 $-x < 2$ $x > -2$
 $x > -2$ ✓

d) $\angle EFH = 55^\circ$ (alternate angles, $AB \parallel CD$)
 $\angle FEH = 55^\circ$ (base angles of isosceles \triangle)
 $x^\circ + 90^\circ + 55^\circ = 180^\circ$ (adjacent angles on a line)
 $x = 35$. ✓✓✓ or straight line.

e) $\frac{4}{7} = \frac{48}{84}$ and $\frac{7}{12} = \frac{49}{84}$ so $\frac{4}{7} < \frac{7}{12}$
 $\frac{3}{5} = \frac{36}{60}$ and $\frac{7}{12} = \frac{35}{60}$ so $\frac{7}{12} < \frac{3}{5}$
 So from lowest to highest, the numbers are:
 $\frac{4}{7}, \frac{7}{12}$ and $\frac{3}{5}$. ✓✓

12

a) $4x - 3(2-x) = 8 - 5(x+3)$

$$4x - 6 + 3x + 8 - 5x - 15 \quad \checkmark$$

$$7x - 6 = -5x - 7 \quad \checkmark$$

$$12x = -1$$

$$x = -\frac{1}{12} \quad \checkmark$$

b) i) $125 - x$ or $\frac{2}{5}(200 - x) \quad \checkmark$

ii) $125 - x = \frac{2}{5}(200 - x) \quad \checkmark$

$$5(125 - x) = 2(200 - x)$$

$$625 - 5x = 400 - 2x$$

$$-3x = -225$$

$$x = 75 \quad \checkmark$$

So 75 marbles were removed

c) i) $\angle ACB = \beta$ (base angles of isosceles \triangle)

$$\alpha + 2\beta = 180^\circ$$
 (angle sum of a \triangle)
$$\alpha = 180^\circ - 2\beta \quad \checkmark \checkmark$$

ii) $180^\circ - 2\beta > 90^\circ$

$$-2\beta > -90^\circ$$

$$2\beta < 90^\circ$$

$$\beta < 45^\circ \quad \checkmark \quad \text{or} \quad 0 < \beta < 45^\circ$$

d) $\frac{2}{H} = \frac{1}{2} + \frac{1}{3} \quad \checkmark$

$$\frac{2}{H} = \frac{5}{6} \quad \checkmark$$

$$\frac{H}{2} = \frac{6}{5}$$

$$H = \frac{12}{5} \quad \text{or} \quad H = 2\frac{2}{5}. \quad \checkmark$$

Q8 a) Let the smallest angle be α
 So the largest angle is 2α

$$\frac{\alpha + 2\alpha}{2} = \frac{3\alpha}{2}$$

So the third angle is $\frac{3\alpha}{2}$

$$\alpha + 2\alpha + \frac{3\alpha}{2} = 180^\circ \text{ (angle sum of a } \Delta) \quad \checkmark$$

$$3\alpha + \frac{3\alpha}{2} = 180^\circ$$

$$6\alpha + 3\alpha = 360^\circ$$

$$9\alpha = 360^\circ$$

$$\alpha = 40^\circ$$

So the angles are $40^\circ, 60^\circ$ and 80° \checkmark

b) $F = \frac{9}{5}C + 32$

Let $F = C$

$$C = \frac{9}{5}C + 32$$

$$C - \frac{9}{5}C = 32$$

$$C(1 - \frac{9}{5}) = 32$$

$$C(-\frac{4}{5}) = 32 \quad \checkmark$$

$$C = 32 \times -\frac{5}{4}$$

$$= -40 \quad \checkmark$$

So the temperature where the two scales have the same value is
 -40°C and -40°F

c) Let the original mass be m tonnes

$$m \times 80\% \times 80\% = 3.6$$

$$m \times \frac{4}{5} \times \frac{4}{5} = 3.6$$

$$m = \frac{3.6 \times \frac{5}{4} \times \frac{5}{4}}{\downarrow}$$

$$= 4.5 \times \frac{5}{4}$$

$$= 5.625$$

So the original mass was 5.625 tonnes

d) Find the prime factorisation of 11111.

37037
7 5291
11 481
13 37

So the children's ages are 3, 7, 11 and 13 years ✓

So the difference in age between the oldest and youngest child is 10 years ✓

12