



CALCULATOR ALLOWED



Advanced level questions



Mini Test 27: Angles

1 A regular octagon is cut into three pieces by drawing lines parallel to the top and bottom as shown in the diagram.

What is the value of x ?

2 A polygon has two angles of 90° and its remaining angles are all 135° . What type of polygon is it?

A pentagon B hexagon
C octagon D decagon

3 Lines l and m are parallel.

What is the value of x ?

A 120 B 130
C 140 D 150

4 A polygon has three angles of 100° and two other equal angles. What size is each of those two remaining angles?

5 Lines l and m are parallel.

What is the value of x ?

A 100 B 120 C 140 D 160

6 The diagram shows a square, an equilateral triangle and an octagon.

What is the value of x ?

7 Lines m and n are parallel.

What is the value of x ?

A 80 B 90 C 100 D 120

8 What is the value of a ?

9 $AB = CB$
 $\angle ABC = 30^\circ$

What is the size of $\angle BCD$?

10 Lines l, m and n are all parallel.

What is the value of x ?

11 The clock shows that it is seven o'clock. What is the size of the obtuse angle formed by the hands?

12 A trapezium is divided into two parts. All the angles are marked.

Which is not necessarily equal to 180° ?

A $a + b + f$ B $a + f + e$
C $a + b + c$ D $b + c + d$

13 Each of the angles of a regular polygon is 140° .

How many sides does the polygon have?

A 6 B 7 C 8 D 9

14 $AD = BD$
 $BC = DC$
 BC is parallel to AD .
 $\angle BAD = 50^\circ$

What is the size of $\angle BCD$?

A 20° B 30° C 50° D 80°

15 Lines l and m are parallel.

What is the value of x ?

16 Which could **not** be placed beside this regular hexagon to fill the space at P?

A two regular hexagons
B four equilateral triangles
C two squares and an equilateral triangle
D one regular octagon and one square

1 45 2 B 3 D 4 120° 5 B 6 75 7 A 8 115

9 105° 10 60 11 150° 12 C 13 D 14 A

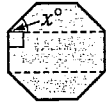
15 60 16 D

1 Angle sum of an octagon = $(8 - 2) \times 180^\circ$
 $= 6 \times 180^\circ$
 $= 1080^\circ$

Each angle of a regular octagon = $1080^\circ \div 8$
 $= 135^\circ$

The quadrilateral formed inside the octagon is a rectangle.

Each angle of a rectangle = 90°
 So $x = 135 - 90$
 $= 45$



2 Try each option:

Angle sum of a pentagon = $(5 - 2) \times 180^\circ$
 $= 3 \times 180^\circ$
 $= 540^\circ$

If there were two angles of 90° and three of 135° , angle sum = $2 \times 90^\circ + 3 \times 135^\circ$
 $= 180^\circ + 405^\circ$
 $= 585^\circ$

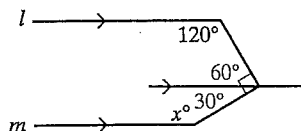
So the polygon is not a pentagon.

Angle sum of a hexagon = $(6 - 2) \times 180^\circ$
 $= 4 \times 180^\circ$
 $= 720^\circ$

If there were two angles of 90° and four of 135° , angle sum = $2 \times 90^\circ + 4 \times 135^\circ$
 $= 180^\circ + 540^\circ$
 $= 720^\circ$

So the polygon is a hexagon.

3 Draw a line through the right angle, parallel to both l and m .



The angle above the line, co-interior to the angle of 120° must be 60° .

[Co-interior angles, formed by parallel lines, add to 180° .]

So the angle below the line is $90^\circ - 60^\circ$ or 30° .

Now $x + 30 = 180$ (co-interior angles, parallel lines)
 $x = 180 - 30$
 $= 150$

4 The polygon has five angles so it is a pentagon.

Angle sum of a pentagon = $(5 - 2) \times 180^\circ$
 $= 3 \times 180^\circ$
 $= 540^\circ$

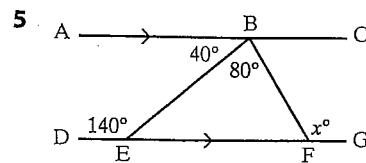
Now there are three angles of 100° .

Sum of 3 angles = $3 \times 100^\circ$
 $= 300^\circ$

Sum of remaining angles = $540^\circ - 300^\circ$
 $= 240^\circ$

But those two angles are equal.

Each angle = $240^\circ \div 2$
 $= 120^\circ$



$\angle ABE + \angle BED = 180^\circ$ (co-interior angles, parallel lines)

So $\angle ABE = 40^\circ$

$\angle ABF = 40^\circ + 80^\circ$
 $= 120^\circ$

$\angle ABF = \angle BFG$ (alternate angles, parallel lines)

So $x = 120$

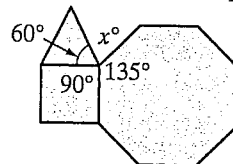
[If you didn't know that the lines were parallel you could still find the answer. The angle beside the one of 140° on the straight line must be 40° and then, because it is the exterior angle of the triangle, $x = 40 + 80$.]

6 Angle sum of an octagon = $(8 - 2) \times 180^\circ$
 $= 6 \times 180^\circ$
 $= 1080^\circ$

Each angle of a regular octagon = $1080^\circ \div 8$
 $= 135^\circ$

Each angle of an equilateral triangle is 60° .

Each angle of a square is 90° .



Angles at a point add to 360° .

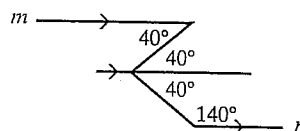
So $x + 135 + 90 + 60 = 360$

$x + 285 = 360$

$x = 360 - 285$

$x = 75$

7 Draw a line through the unknown angle, parallel to both m and n .



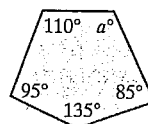
Because alternate angles formed by parallel lines are equal, the top part of the unknown angle is 40° .

Because co-interior angles formed by parallel lines add to 180° , the bottom part of the unknown angle is 40° .

So $x = 40 + 40$
 $= 80$

8 The shape is a pentagon.

Angle sum of a pentagon = $(5 - 2) \times 180^\circ$
 $= 3 \times 180^\circ$
 $= 540^\circ$

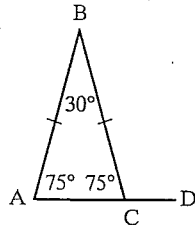


$$\begin{aligned} \text{So } a + 85 + 135 + 95 + 110 &= 540 \\ a + 425 &= 540 \\ a &= 540 - 425 \\ &= 115 \end{aligned}$$

- 9 $\angle BAC + \angle ABC + \angle BCA = 180^\circ$ (angle sum of a triangle)

Now $\angle ABC = 30^\circ$,
 so $\angle BAC + \angle BCA = 150^\circ$
 But $\angle BAC = \angle BCA$ (angles opposite equal sides in an isosceles triangle)

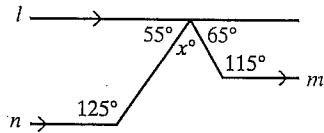
$$\begin{aligned} \text{So } \angle BCA &= 150^\circ \div 2 \\ &= 75^\circ \end{aligned}$$



$\angle BCD + \angle BCA = 180^\circ$ (angles in a straight line)

$$\begin{aligned} \text{So } \angle BCD &= 180^\circ - 75^\circ \\ &= 105^\circ \end{aligned}$$

- 10 Because co-interior angles formed by parallel lines add to 180° , the angle on the left at the top is 55° , and the angle on the right is 65° .



Now angles in a straight line add to 180° .
 So $x + 55 + 65 = 180$
 $x + 120 = 180$
 $x = 180 - 120$
 $= 60$

- 11 A clock face is divided into 12 parts.

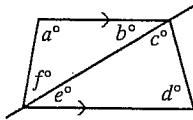
So each part is $360^\circ \div 12 = 30^\circ$

There are five parts in the obtuse angle between the hands at seven o'clock.

$$\begin{aligned} \text{Angle} &= 5 \times 30^\circ \\ &= 150^\circ \end{aligned}$$



- 12 Consider the options:



$a + b + f = 180$ (angle sum of a triangle)

$a + f + e = 180$ (co-interior angles, parallel lines)

$a + b + c$ is not necessarily equal to 180.

$b + c + d = 180$ (co-interior angles, parallel lines)

The expression that is not necessarily equal to 180 is $a + b + c$.

[If the other pair of sides of the trapezium were parallel (i.e. the trapezium was in fact a parallelogram) then $a + b + c$ would equal 180. If those sides are not parallel, then $a + b + c$ would not equal 180.]

- 13 Consider the options.

If the polygon had 6 sides then:
 angle sum = $(6 - 2) \times 180^\circ$
 $= 4 \times 180^\circ$
 $= 720^\circ$

Each angle = $720^\circ \div 6$
 $= 120^\circ$

So the polygon does not have 6 sides.

If the polygon had 7 sides then:
 angle sum = $(7 - 2) \times 180^\circ$
 $= 5 \times 180^\circ$
 $= 900^\circ$

Each angle = $900^\circ \div 7$
 $= 128.57\dots^\circ$

So the polygon does not have 7 sides.

If the polygon had 8 sides then:
 angle sum = $(8 - 2) \times 180^\circ$
 $= 6 \times 180^\circ$
 $= 1080^\circ$

Each angle = $1080^\circ \div 8$
 $= 135^\circ$

So the polygon does not have 8 sides.

If the polygon had 9 sides then:
 angle sum = $(9 - 2) \times 180^\circ$
 $= 7 \times 180^\circ$
 $= 1260^\circ$

Each angle = $1260^\circ \div 9$
 $= 140^\circ$

So the polygon does have 9 sides.

- 14 $\angle ABD = \angle BAD$ (opposite equal sides, isosceles triangle)

So $\angle ABD = 50^\circ$

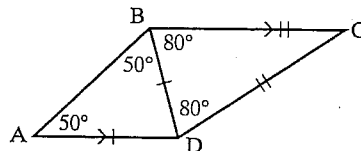
$\angle ABC + \angle BAD = 180^\circ$ (co-interior angles, parallel lines)

So $\angle ABC = 130^\circ$

$\angle CBD = 130^\circ - 50^\circ$
 $= 80^\circ$

$\angle CDB = \angle CBD$ (angles opposite equal sides, isosceles triangle)

So $\angle CDB = 80^\circ$

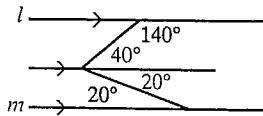


$\angle CBD + \angle CDB + \angle BCD = 180^\circ$ (angle sum of a triangle)

$80^\circ + 80^\circ + \angle BCD = 180^\circ$

$$\begin{aligned} \angle BCD &= 180^\circ - 160^\circ \\ &= 20^\circ \end{aligned}$$

- 15 Draw a line through the unknown angle, parallel to both l and m .



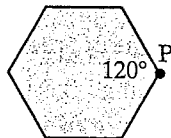
Because co-interior angles formed by parallel lines add to 180° , the top part of the unknown angle is 40° .

Because alternate angles formed by parallel lines are equal, the bottom part of the unknown angle is 20° .

$$\begin{aligned} \text{So } x &= 40 + 20 \\ &= 60 \end{aligned}$$

- 16 Angle sum of a hexagon = $(6 - 2) \times 180^\circ$
 $= 4 \times 180^\circ$
 $= 720^\circ$

$$\begin{aligned} \text{Each angle} &= 720^\circ \div 6 \\ &= 120^\circ \end{aligned}$$



Now angles at a point add to 360° .

$$\begin{aligned} \text{Remaining angle} &= 360^\circ - 120^\circ \\ &= 240^\circ \end{aligned}$$

Now try the options.

Two regular hexagons:

$$2 \times 120^\circ = 240^\circ$$

So two regular hexagons could fill the space.

Four equilateral triangles:

Each angle of an equilateral triangle is 60° .

$$4 \times 60^\circ = 240^\circ$$

So four equilateral triangles could fill the space.

Two squares and an equilateral triangle:

Each angle of a square is 90°

$$\begin{aligned} 2 \times 90^\circ + 60^\circ &= 180^\circ + 60^\circ \\ &= 240^\circ \end{aligned}$$

So two squares and an equilateral triangle could fill the space.

One regular octagon and one square:

$$\begin{aligned} \text{Angle sum of an octagon} &= (8 - 2) \times 180^\circ \\ &= 6 \times 180^\circ \\ &= 1080^\circ \end{aligned}$$

$$\begin{aligned} \text{Each angle of a regular octagon} &= 1080^\circ \div 8 \\ &= 135^\circ \end{aligned}$$

$$135^\circ + 90^\circ = 225^\circ$$

So a regular octagon and a square will not fill the space.