Year 9 Mathematics

Common Task

Term 3, 2006

Name: _____

Class Teacher (circle) ***RBL*** ***GP*** ***MWA***

Topics:
Surds
Recurring Decimals
Properties of Geometric Figures
Circle Geometry
Coordinate Geometry

Time: ***50 minutes***

Instructions:
Answer all questions.

Calculators may be used.

Marks may not be awarded for untidy or careless work.

Show all necessary working.

Diagrams are NOT drawn to scale

Marks

1. Simplify the following surds:

(a) $2\sqrt{3}+5\sqrt{3}-4\sqrt{2}$ 1

(b) $\sqrt{20}+\sqrt{45}-\sqrt{80}$ 1

(c) $\left(3-2\sqrt{5}\right)^2$ 2

2. Rationalise the denominator in the following:

(a) $\dfrac{\sqrt{3}}{\sqrt{5}}$ 1

(b) $\dfrac{6+\sqrt{2}}{4-\sqrt{2}}$ 2

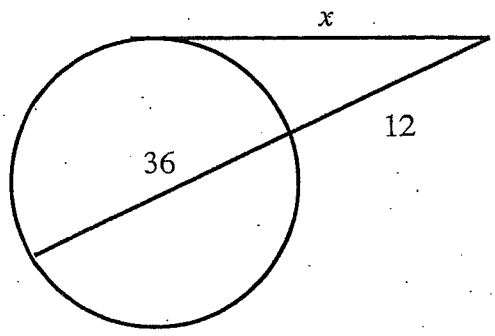
Marks

3. Convert $0.\dot{3}\dot{8}$ to a fraction in its simplest form. 2

4. Show that $0.\dot{9}=1$. 2

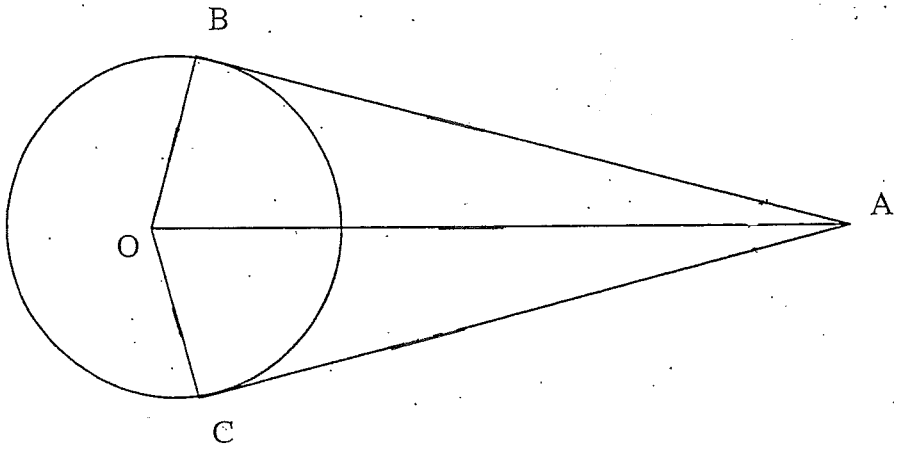
2

5. Find the value of x in the following, giving reasons. 2



Marks

6. AB and AC are tangents to a circle with centre O.

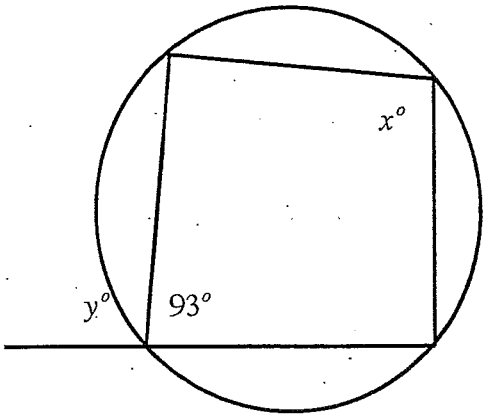


(a) Using your circle geometry rules, prove that $\triangle ABO \equiv \triangle ACO$. 2

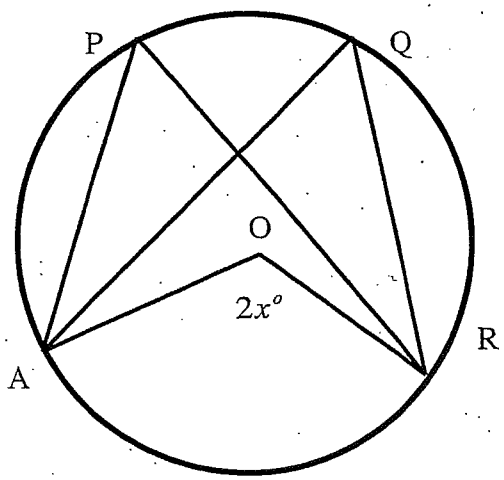
(b) Hence prove that OA bisects angle BAC. 1

Marks

7. Find the value of x and y in the diagram below. Give reasons for your answer. 2



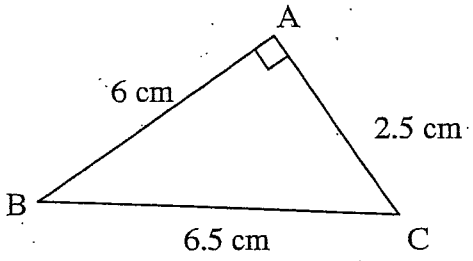
8. In the following diagram, O is the centre of a circle and $\angle AOR = 2x^o$. Find the values of $\angle APR$ and $\angle AQR$ in terms of x. Give reasons for your answers. 2

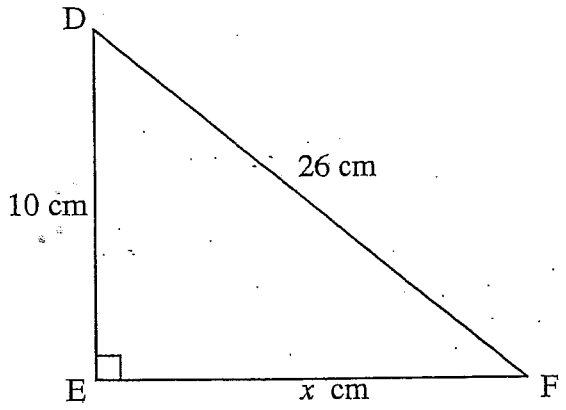


Marks

9. Show that the external angle sum of an octagon is 360°. 2

10.





(a) Prove that $\triangle ABC$ is similar to $\triangle DEF$. 2

(b) Hence find the value of x. 1

Marks

11. Given an interval between the points $A(3, 4)$ and $B(6, -8)$, find showing **all** working:

(a) The midpoint of the interval AB. **1**

(b) The gradient of the interval AB. **2**

(c) The length of the interval AB. **2**

(d) Show that the equation of a line passing through A and B is $y = -4x + 16$ **2**

Marks

12. Find the equation of a line perpendicular to the line $y = -4x + 16$ and passing through the point $P(1, 1)$. Write your answer in general form. 3

13. Complete the following sentences:

(a) $y = 0$ is the equation of __ 1

(b) $y = 4 - \frac{x}{4}$ has a y-intercept of 1

(c) $x = 4$ is a line parallel to which axis? 1

(d) $y = \frac{3x}{5} + 4$ has a gradient of 1

End of Examination

Marks

1. Simplify the following surds:

(a) $2\sqrt{3}+5\sqrt{3}-4\sqrt{2}$ 1

$= 7\sqrt{3} - 4\sqrt{2}$

(b) $\sqrt{20}+\sqrt{45}-\sqrt{80}$ 1

$= 2\sqrt{5} + 3\sqrt{5} - 4\sqrt{5}$

$= \sqrt{5}$

(c) $(3-2\sqrt{5})^2$ 2

$= (3)^2 - 2\times 3\times 2\sqrt{5} + (-2\sqrt{5})^2$

$= 9 - 12\sqrt{5} + 20$

$= 29 - 12\sqrt{5}$

2. Rationalise the denominator in the following:

(a) $\frac{\sqrt{3}}{\sqrt{5}}$ 1

$= \frac{\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{\sqrt{15}}{5}$

(b) $\frac{6+\sqrt{2}}{4-\sqrt{2}}$ 2

$= \frac{6+\sqrt{2}}{4-\sqrt{2}} \times \frac{4+\sqrt{2}}{4+\sqrt{2}}$

$= \frac{(6+\sqrt{2})(4+\sqrt{2})}{4^2-(\sqrt{2})^2}$

$= \frac{24+6\sqrt{2}+4\sqrt{2}+2}{16-2}$

$= \frac{26+10\sqrt{2}}{14}$

$= \frac{13+5\sqrt{2}}{7}$

Marks

3. Convert $0.\dot{3}\dot{8}$ to a fraction in its simplest form. 2

Let $n = 0.383838\ldots$

$100n = 38.383838\ldots$

$99n = 38$

$n = \frac{38}{99}$

$\therefore 0.\dot{3}\dot{8} = \frac{38}{99}$

4. Show that $0.\dot{9}=1$. 2

Let $n = 0.99999\ldots$

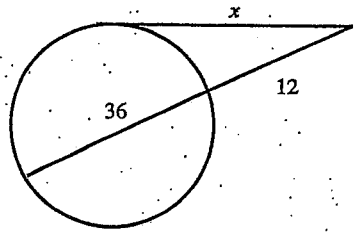
$10n = 9.9999\ldots$

$9n = 9$

$n = 1$

$\therefore 0.\dot{9} = 1$

5. Find the value of x in the following, giving reasons. 2



$x^2 = 12 \times (36+12)$

$= 12 \times 48$

$= 576$

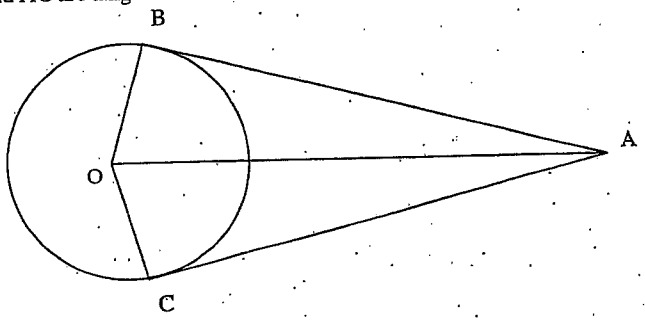
$x = \sqrt{576}$

$= 24$

- The square of the length of the tange to a circle from an external poin equals the product of the intercepts of any secant from the point.

Marks

6. AB and AC are tangents to a circle with centre O.



(a) Using your circle geometry rules, prove that $\triangle ABO \equiv \triangle ACO$. 2

$OB = OC$ (radii of circle)

$\angle OBA = \angle OCA = 90°$ (tangent to a circle is perpendicular to the radius drawn to the point of contact)

OA is a common side and is the hypotenuse of $\triangle ACO$ and of $\triangle ABO$

$\therefore \triangle ABO \equiv \triangle ACO$ (RHS)

(b) Hence prove that OA bisects angle BAC. 1

$\angle BAO = \angle CAO$ (Corresponding angles in congruent $\triangle$s)

$\angle BAC = \angle BAO + \angle CAO$

$= \angle BAO + \angle BAO$

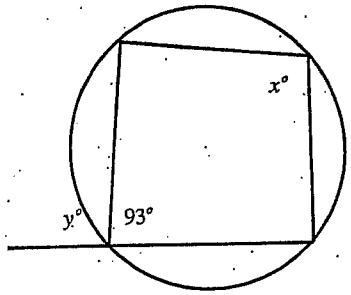
$= 2\angle BAO$

$\therefore \angle BAO = \frac{1}{2}\angle BAC = \angle CAO$

$\therefore$ OA bisects $\angle BAC$

Marks

7. Find the value of x and y in the diagram below. Give reasons for your answer. 2



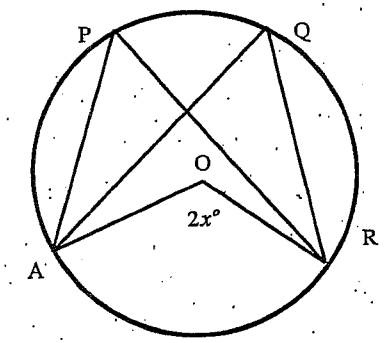
$x = 180° - 93°$ - Opposite angles of a cyclic quadrilateral are supplementary

$= 87°$

$y = x = 87°$ - Exterior angle of a cyclic quadrilateral is equal to the interior opposite angle

OR $y = 180 - 93 = 87°$ - Straight line has 180°

8. In the following diagram, O is the centre of a circle and $\angle AOR = 2x°$. Find the values of $\angle APR$ and $\angle AQR$ in terms of x. Give reasons for your answers. 2



$\angle APR = x°$ - Angle at the centre of a circle is two times the angle at the circumference standing on same arc.

$\angle AQR = x°$ - Same reason as above

OR Angles at the circumference of a circle on the same arc are equal.

Marks

9. Show that the external angle sum of an octagon is 360°. 2

At each vertex of the octagon, the sum of the interior and exterior angles is 180° (a straight line).

The sum of interior angles is given by $(n-2)\times 180° = (8-2)\times 180 = 1080°$

Sum of Exterior Angles = Sum of all interior and exterior angles − sum of interior angles.

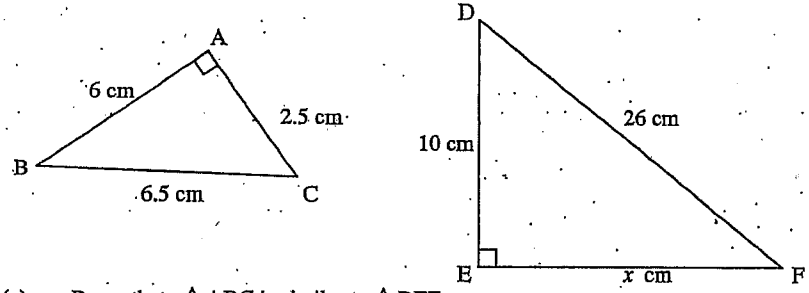
$= 8 \times 180° - 1080°$

$= 1440° - 1080°$

$= 360°$

∴ External angle sum of an octagon is 360°.

10.



(a) Prove that △ABC is similar to △DEF. 2

For similarity, the ratio of corresponding sides must be equal.

Hypotenuse − $\frac{BC}{DF} = \frac{6.5}{26} = \frac{1}{4} = 0.25$ − hypotenuse and one other side

Other Side − $\frac{AC}{DE} = \frac{2.5}{10} = \frac{1}{4} = 0.25$

$\therefore \frac{BC}{DF} = \frac{AC}{DE}$ ∴ △ABC ||| △DEF (RHS)

(b) Hence find the value of x. 1

$\frac{AC}{DE} = \frac{AB}{EF}$

$\frac{2.5}{10} = \frac{6}{x}$

$2.5x = 60$

$x = 24$ cm

Marks

11. Given an interval between the points $A(3, 4)$ and $B(6, -8)$, find showing all working:

(a) The midpoint of the interval AB. 1

$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{3+6}{2}, \frac{4+-8}{2}\right)$

$= \left(\frac{9}{2}, \frac{-4}{2}\right)$

$= \left(4\frac{1}{2}, -2\right)$

(b). The gradient of the interval AB. 2

$m = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-8-4}{6-3}$

$= \frac{-12}{3}$

$= -4$

(c) The length of the interval AB. 2

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(6-3)^2 + (-8-4)^2}$

$= \sqrt{3^2 + (-12)^2}$

$= \sqrt{9 + 144}$

$= \sqrt{153}$

$= 12.37$ (2 d.p)

(d) Show that the equation of a line passing through A and B is $y = -4x + 16$

$y - y_1 = m(x - x_1)$

$y - 4 = -4(x - 3)$

$y - 4 = -4x + 12$

$y = -4x + 16$

Marks

12. Find the equation of a line perpendicular to the line $y = -4x + 16$ and passing through the point $P(1, 1)$. Write your answer in general form. 3

For perpendicular lines : $m_1 = \frac{-1}{m_2}$

$\therefore$ Gradient of perpendicular line is $\frac{-1}{-4} = \frac{1}{4}$

Equation of Line: $y - y_1 = m(x - x_1)$

$y - 1 = \frac{1}{4}(x - 1)$

$4y - 4 = x - 1$

$x - 4y + 3 = 0,$

$\therefore$ Equation of line perpendicular to $y = -4x + 16$ passing through $P(1, 1)$ is

$x - 4y + 3 = 0$

13. Complete the following sentences:

(a) $y = 0$ is the equation of the x-axis. 1

(b) $y = 4 - \frac{x}{4}$ has a y-intercept of 4 or $(0, 4)$. 1

(c) $x = 4$ is a line parallel to which axis? the y-axis 1

(d) $y = \frac{3x}{5} + 4$ has a gradient of $\frac{3}{5}$. 1

End of Examination