Topic 8: Exercises on the Ellipse
Level 1

1. For the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ find (a) the eccentricity, (b) the coordinates of the foci, (c) the equations of the directrices. Sketch the ellipse.

(a) $\frac{3}{5}$; (b) $(\pm 3,0)$; (c) $x=\pm\frac{25}{3}$

2. For the ellipse $\frac{x^2}{3}+\frac{y^2}{2}=1$ find (a) the eccentricity, (b) the coordinates of the foci, (c) the equations of the directrices. Sketch the ellipse.

(a) $\frac{1}{\sqrt{3}}$; (b) $(\pm 1,0)$; (c) $x=\pm 3$.

3. The ellipse has eccentricity $\frac{4}{5}$ and foci $(-4,0)$ and $(4,0)$. Find the equation of this ellipse.

$$\boxed{\frac{x^2}{25}+\frac{y^2}{9}=1}$$

4. A variable point $P(x,y)$ moves so that its distance from $(1,0)$ is one-third its distance from $x=9$. Find the locus of P.

$$\boxed{\frac{x^2}{9}+\frac{y^2}{8}=1}$$

5. An ellipse has its center at the origin and its foci on the x-axis. The distance between the foci is 4 units and the distance between the directrices is 16 units. Find the equation of the ellipse.

$$\frac{x^2}{16}+\frac{y^2}{12}=1$$

6. A point P lies on the ellipse $\frac{x^2}{9}+\frac{y^2}{8}=1$ with foci S and S'. Find PS' if $PS=2$.

$$PS'=4$$

7. Find the parametric equation of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

(a) $x = 4\cos\theta,\ y = 3\sin\theta,\ -\pi < \theta \le \pi$

8. Find the Cartesian equation of the ellipse $x = 3\cos\theta,\ y = 2\sin\theta$;

$\frac{x^2}{9}+\frac{y^2}{4}=1$

9. The points $P\,(a\cos\theta, b\sin\theta)$ and $Q\,[a\cos(\pi+\theta), b\sin(\pi+\theta)]$ lie on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Show that the chord PQ passes through $(0,0)$.

10. The points $P\,(a\cos\theta, b\sin\theta)$ and $Q\,[a\cos(-\theta), b\sin(-\theta)]$ are the extremities of the latus rectum $x = ae$ of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Show that (a) $\cos\theta = e$; (b) PQ has length $2\frac{b^2}{a}$.

11. The points $P\,(a\cos\theta, b\sin\theta)$ and $Q\,(a\cos\phi, b\sin\phi)$ lie on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the chord PQ subtends a right angle at $(0,0)$. Show that $\tan\theta\tan\phi=-\frac{a^2}{b^2}$.

12. Find the equations of the tangent and the normal to the ellipse $\frac{x^2}{15}+\frac{y^2}{10}=1$ at the point $(3,2)$.

$x+y=5$; $x-y=1$

13. Find the equations of the tangent and the normal to the ellipse $x = 6\cos\theta, y = 2\sin\theta$ at the point where $\theta = \frac{\pi}{6}$.

$\sqrt{3}x + 3y = 12$; $3x - \sqrt{3}y = 8\sqrt{3}$

14. Find the equation of the chord of contact of tangents from the point $(5,4)$ to the ellipse $\frac{x^2}{15} + \frac{y^2}{10} = 1$.

$5x + 6y = 15$

15. The point $P\,(a\cos\theta, b\sin\theta)$ lies on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. The normal at P cuts the x-axis at X and the y-axis at Y. Show that $\frac{PX}{PY}=\frac{b^2}{a^2}$.