Topic 8: Exercises on the Ellipse
Level 3

1. Points $P(a\cos\theta, b\sin\theta)$ and $Q(a\cos\phi, b\sin\phi)$ lie on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Find the equation of the chord PQ. Hence show that if PQ subtends a right angle at the point $A(a,0)$ then PQ passes through a fixed point on the x-axis.

$$\frac{x}{a}\cos\left(\frac{\theta+\phi}{2}\right)+\frac{y}{b}\sin\left(\frac{\theta+\phi}{2}\right)=\cos\left(\frac{\theta-\phi}{2}\right)$$

2. Find the equation of the tangent and normal to (a) the ellipse $\frac{x^2}{8}+\frac{y^2}{2}=1$ at the point $(2,1)$; (b) the ellipse $x=4\cos\theta, y=2\sin\theta$ at the point where $\theta=\frac{\pi}{3}$.

(a) $x+2y=4$, $2x-y=3$; (b) $x+2\sqrt{3}y=8$, $6x-\sqrt{3}y=9$

3. The point $P(a\cos\theta, b\sin\theta)$ lies on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. The points T and T' are the feet of the perpendiculars from the foci S and S' respectively to this tangent. Show that $ST \cdot S'T' = b^2$.

4. The point $P(a \cos \theta, b \sin \theta)$ lies on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The normal at the P cuts the x-axis at G, and N is the foot of the perpendicular from P to the x-axis. Show that $SG = eSP$, and $S'G = eS'P$.

5. Show that the chord of contact of the tangents from the point $P_0(x_0, y_0)$ to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ has equation $\frac{x_0 x}{a^2}+\frac{y_0 y}{b^2}=1$.

6. Write down the equation of the chord of contact of the tangents from the point $(4,-1)$ to the ellipse $x^2+2y^2=6$. Hence find the coordinates of the points of contact and the equations of these tangents.

$2x-y=3$; $\left(\frac{2}{3},-\frac{5}{3}\right)$, $x-5y=9$; $(2,1)$, $x+y=3$

7. Find the equations and the coordinates of the points of contact of the tangents to $x^2 + 2y^2 = 19$ which are parallel to $x + 6y = 5$.

$x + 6y = 19$, (1,3); $x + 6y = -19$, (−1,−3)

8. Find the equations and the coordinates of the points of contact of the tangents to $8x^2 + 3y^2 = 35$ from the point $\left(\frac{5}{4}, 5\right)$.

$16x + 3y = 35$, $(2,1)$; $-8x + 9y = 35$, $(-1,3)$

9. The point $P(a\cos\theta, b\sin\theta)$ lies on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a>b>0$. The tangent and the normal at P cut the y-axis at A and B respectively, and S is a focus of the ellipse. (i) Show that $ASB = 90^0$. (ii) Hence show that A, P, S and B are concyclic and state the location of the center of the circle through A, P, S and B.