Topic 9: Exercises on the Hyperbola
Level 1

1. For the hyperbola $\frac{x^2}{9}-\frac{y^2}{16}=1$ find (a) the eccentricity, (b) the coordinates of the foci, (c) the equations of the directrices, (d) the equation of the asymptotes. Sketch the hyperbola.

(a) $\frac{5}{3}$; (b) $(\pm 5,0)$; (c) $x=\pm\frac{9}{5}$; (d) $y=\pm\frac{4}{3}x$

2. For the hyperbola $\frac{x^2}{2}-\frac{y^2}{4}=1$ find (a) the eccentricity, (b) the coordinates of the foci, (c) the equations of the directrices, (d) the equation of the asymptotes. Sketch the hyperbola.

(a) $\sqrt{3}$; (b) $(\pm\sqrt{6},0)$; (c) $x=\pm\sqrt{\frac{2}{3}}$; (d) $y=\pm\sqrt{2}x$

3. The hyperbola has eccentricity $\frac{3}{2}$ and directrices $x=-4$ and $x=4$. Find the equation of this hyperbola.

$$\frac{x^2}{36}-\frac{y^2}{45}=1$$

4. A variable point $P(x,y)$ moves so that its distance from $(9,0)$ is three times its distance from $x=1$. Find the locus of P.

$$\frac{x^2}{9}-\frac{y^2}{72}=1$$

5. A point P lies on the hyperbola $\frac{x^2}{9}-\frac{y^2}{72}=1$ with foci S and S'. Find PS' if $PS=2$.

$PS'=8$

6. Find the parametric equation of the hyperbola $\frac{x^2}{16}-\frac{y^2}{25}=1$.

$x=4\sec\theta,\ y=5\tan\theta,\ -\pi<\theta\le\pi,\theta\neq\pm\frac{\pi}{2}$

7. Find the Cartesian equation of the hyperbola $x = 3\sec\theta$, $y = 4\tan\theta$.

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

8. The points $P(a\sec\theta, b\tan\theta)$ and $Q(a\sec\phi, b\tan\phi)$ lie on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and the chord PQ subtends a right angle at $(0,0)$. Show that $\sin\theta\sin\phi = -\frac{a^2}{b^2}$.

9. Find the equations of the tangent and the normal to the hyperbola $\frac{x^2}{6}-\frac{y^2}{8}=1$ at the point $(3,2)$.

$2x-y=4;\ x+2y=7$

10. Find the equations of the tangent and the normal to the hyperbola $x=2\sec\theta, y=3\tan\theta$ at the point where $\theta=\frac{\pi}{3}$.

$3x-\sqrt{3}y=3;\ x+\sqrt{3}y=13$

11. Find the equation of the chord of contact of tangents from the point $(1,2)$ to the hyperbola $\frac{x^2}{6}-\frac{y^2}{8}=1$.

$2x-3y=12$

12. S is a focus of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. The tangent at $(a,0)$ meets an asymptote at the point T. Show that $OT=OS$.

13. Find the equation of the chord of contact of the tangents from the point $(1,3)$ to the hyperbola $\frac{x^2}{4}-\frac{y^2}{15}=1$. Hence find the coordinates of their points of contact and the equations of these tangents.

$5x-4y=20$; $y=-4x+7$, $\left(\frac{16}{7},-\frac{15}{7}\right)$; $y=2x+1$, $(-8,-15)$

14. Show that the chord of contact of the tangents from a point on a directrix of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is a focal chord through the corresponding focus.