Topic 9: Exercises on the Hyperbola
Level 2

1. For the hyperbola $x^2 - y^2 = 4$, find (a) the eccentricity, (b) the coordinates of the foci, (c) the equations of the directrices, (d) the equations of the asymptotes. Sketch the hyperbola.

(a) $\sqrt{2}$; (b) $(\pm 2\sqrt{2}, 0)$; (c) $x = \pm\sqrt{2}$; (d) $y = \pm x$

2. The hyperbola has eccentricity $\frac{5}{4}$ and foci $(-5,0)$ and $(5,0)$. Find the equation of the hyperbola.

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

3. A point P lies on the hyperbola $\frac{x^2}{9}-\frac{y^2}{72}=1$ with foci S and S'. Find PS' if $PS=8$.

14 or 2

4. A hyperbola has center at the origin and foci on the x-axis. The distance between the foci is 16 units and the distance between the directrices is 4 units. Find the equation of the hyperbola.

$\frac{x^2}{16}-\frac{y^2}{48}=1.$

5. Show that the equation $\frac{x^2}{29-k}+\frac{y^2}{4-k}=1$, where k is a real number, represents (i) a hyperbola if $4<k<29$. Show that the foci of the hyperbola are independent of the value of k.

6. Find the parametric equations of the hyperbola $x^2-y^2=4$.

$x=2\sec\theta,\ y=2\tan\theta$

7. Find the Cartesian equations of the hyperbola $x = 2\sec\theta$, $y = 5\tan\theta$.

$$\frac{x^2}{4} - \frac{y^2}{25} = 1$$

8. The points $P(a\sec\theta, b\tan\theta)$ and $Q[a\sec(\pi-\theta), b\tan(\pi-\theta)]$ lie on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Show that PQ passes through $(0,0)$.

9. The points $P(a\sec\theta, b\tan\theta)$ and $Q(a\sec\phi, b\tan\phi)$ lie on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If PQ subtends a right angle at $(a, 0)$. Show that $\tan\frac{\theta}{2}\tan\frac{\phi}{2}=-\frac{b^2}{a^2}$.

10. The points $P(a\sec\theta, b\tan\theta)$ and $Q[a\sec(-\theta), b\tan(-\theta)]$ are the extremeties of the latus rectum $x = ae$ of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Show that (a) $\sec\theta = e$; (b) PQ has length $\frac{2b^2}{a}$.

11. Find the equation of the tangent and the normal to the hyperbola $9x^2 - 2y^2 = 18$ at the point $(2,-3)$.

$3x + y = 3$, $x - 3y = 11$

12. Find the equation of the tangent and the normal to the hyperbola $x = 2\sec\theta, y = 4\tan\theta$ at the point where $\theta = -\frac{\pi}{4}$.

$2\sqrt{2}x + y = 4,\ x - 2\sqrt{2}y = 10\sqrt{2}$

13. Find the equation of the chord of contact of tangents to the hyperbola $9x^2 - 2y^2 = 18$ from the point $(1,2)$.

$9x - 4y = 18$

14. The point $P(a\sec\theta, b\tan\theta)$ lies on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. The tangent at P cuts the x-axis at X and the y-axis at Y. Show that $\frac{PX}{PY}=\sin^2\theta$ and deduce that if P is an extremity of a latus rectum, then $\frac{PX}{PY}=\frac{e^2-1}{e^2}$.