Topic 20: Exercises on Mechanics 1
Level 1

1. A particle moves in a straight line with acceleration which is inversely proportional to t^3, where t is the time. The particle has a velocity of 3 ms^{-1} when $t = 1$ and its velocity approaches a limiting value of 5 ms^{-1}. Find an expression for its velocity at time t.

$$v = 5 - \frac{2}{t^2}$$

2. A particle moves in a straight line away from a fixed point O in the line, such that when its distance from O is x its speed v is given by $v = \frac{k}{x}$, for some constant k. Show that the particle has a retardation which is inversely proportional to x^3.

3. A particle moves in a straight line away from a fixed point O in the line, such that at time t its displacement from O is x and its displacement from O is x and its velocity is v. At time $t = 0$, $x = 0$, and $v = V$. Subsequently the particle is slowing down at a rate proportional to the square of its speed. Find expressions for the velocity v in terms of the displacement x.

$v = Ve^{-kx}$

4. A particle moves in a straight line away from a fixed point O in the line, such that at time t its displacement from O is x and its velocity v is given by $\frac{1}{v} = A + Bt$, for some positive constants A and B. Show that the retardation of the particle is proportional to the square of the speed.

5. A particle moves in a straight line away from a fixed point O in the line, such that at time t its displacement from O is x and its velocity is v. At time $t = 0$, $x = 0$, and $v = V$. Subsequently the particle is slowing down at a rate is equal to kv^3, where k is a positive constant. Show that $kx = \frac{1}{v} - \frac{1}{V}$.

6. A particle moves in a straight line away from a fixed point O in the line such that at time t its displacement from O is x and its velocity is v. At time $t = 0$, $x = 0$, and $v = 1$. Subsequently the particle experiences a retardation of magnitude e^v. Find the distance traveled by the particle in coming to rest.

$1 - 2e^{-1}$

7. A particle of mass m moves in a straight line away from a fixed point O in the line, such that at time t its displacement from O is x and its velocity is v. At time $t = 0$, $x = 1$, and $v = 0$. Subsequently the only force acting on the particle is one of magnitude $m\frac{k}{x^2}$, where k is a positive constant, in a direction away from O. Show that v can not exceed $\sqrt{(2k)}$.

8. A particle of mass m moves in a horizontal straight line. The only force acting on the particle is a resistance of magnitude mkv^3 where v is its speed and k is a positive constant. At time t the distance from a fixed point on the line is x. When $t = 0$, $x = 0$ and $v = u$. Show that $v = \dfrac{u}{\sqrt{(1+2ku^2t)}} = \dfrac{u}{1+kux}$.

9. The force of attraction experienced by a particle of mass m at a distance $x(>r)$ from the center O of the earth towards O is $\frac{mgr^2}{x^2}$, where r is the radius of the earth. A particle of mass m starts from the surface with speed u directly away from O.
(a) Find the subsequent speed when the particle is distance x from O.

$$v = \sqrt{u^2 - 2gr\left(1 - \frac{r}{x}\right)}$$

(b) Deduce that the particle will escape from the attraction of the earth if $u^2 > 2gr$.

10. A stone of mass m falls freely from rest under gravity for a time $\frac{1}{2k}$. Find the distance it has fallen and the speed attained.

$\frac{g}{8k^2}; \frac{g}{2k}$