

**Topic 20: Exercises on Mechanics 1**

**Level 3**

1. A particle moves in a straight line away from a fixed point  $O$  in the line, such that when its distance from  $O$  is  $x$  its speed  $v$  is given by  $v = \frac{k}{x}$ , for some constant  $k$ . If  $A$ ,  $B$ ,  $C$  and  $D$  are points in that order on the straight line, such that the distances  $AB$ ,  $BC$ , and  $CD$  are all equal, show that the times taken to travel these successive distances in arithmetic progression.

2. A particle of mass  $m$  moves in a straight line away from a fixed point  $O$  in the line such that at time  $t$  its displacement from  $O$  is  $x$  and its velocity is  $v$ . At time  $t = 0$ ,  $x = 1$  and  $v = 0$ . Subsequently the only force acting on the particle is one of magnitude  $m \frac{k}{x^2}$ , where  $k$  is a positive constant, in a direction away from  $O$ . Find the time taken by the particle to move from  $x = 2$  to  $x = 4$ .

$$\frac{1}{2k} \left\{ \ln \frac{2+\sqrt{3}}{2+\sqrt{2}} + 2\sqrt{3} - \sqrt{2} \right\}$$

3. A particle of mass  $0.1 \text{ kg}$  moving on a smooth horizontal table with constant speed  $v \text{ ms}^{-1}$  describes a circle with centre  $O$  and radius  $r \text{ m}$ . The particle is attached towards  $O$  by a force of magnitude  $4v \text{ N}$  and repelled from  $O$  by a force of magnitude  $\frac{k}{r} \text{ N}$  where  $k$  is a constant. If  $r = 1$ , find the set of possible values of  $k$ .

$$0 < k \leq 40$$

4. A particle of mass  $0.2 \text{ kg}$  moving on a smooth horizontal table with constant speed  $v \text{ ms}^{-1}$  describes a circle with centre  $O$  such that  $OP = rm$ . The particle is subject to two forces, one towards  $O$  with magnitude  $8v\text{N}$  and one away from  $O$  with magnitude  $\frac{k}{r^2}\text{N}$  where  $k$  is a positive constant. If the period of revolution is  $\frac{\pi}{5}$  when  $v = 20$ , find the values of  $r$  and  $k$ .

$r = 2, k = 480$
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