

**Topic 23: Exercises on Motion in a Horizontal Circle**

**Level 2**

1. A particle of mass  $0.25 \text{ kg}$  is attached to one end of a light inextensible string of length  $0.5 \text{ m}$ . The other end is fixed to a point  $A$  on a smooth horizontal table. The particle is set in motion in a circular path. If the speed of the particle is  $8 \text{ ms}^{-1}$ , find the tension in the string and the reaction with the table.

$32 \text{ N}, \frac{1}{4}g\text{N}$
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2. A particle of mass  $0.5 \text{ kg}$  is attached to one end of a light inextensible string of length  $2 \text{ m}$ . The other end is fixed to a point  $A$  on a smooth horizontal table. The particle is set in motion in a circular peth. If the string breaks when the tension in it exceeds  $64 \text{ N}$ , find the greatest speed at which the particle can travel.

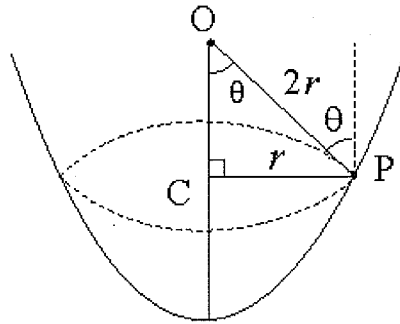
$16 \text{ ms}^{-1}$

3. A mass of  $2 \text{ kg}$  is revolving at the end of a string  $2 \text{ m}$  long on a smooth horizontal table with uniform angular speed. If the string would break under a tension equal to the weight of  $20 \text{ kg}$ , find the greatest positive speed of the mass.

$$\sqrt{20g} \text{ ms}^{-1}$$

4. A particle moves with constant angular velocity  $\omega$  in a horizontal circle of radius  $r$  on the inside of a fixed smooth hemispherical bowl of internal radius  $2r$ . Show that

$$\omega^2 = \frac{g}{r\sqrt{3}}.$$



5. A particle  $P$  of mass  $0.2 \text{ kg}$  moving on a smooth horizontal table with constant speed  $v \text{ ms}^{-1}$  describes a circle with centre  $O$  such that  $OP = r \text{ m}$ . The particle is subject to two forces, one towards  $O$  with magnitude  $8v \text{ N}$  and one away from  $O$  with magnitude  $\frac{k}{r^2} \text{ N}$ , where  $k$  is a positive constant. If  $r = 1$ , find the set of possible values of  $k$ .

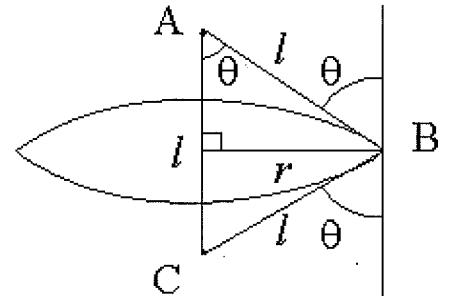
$$0 \leq k \leq 80$$

6. A particle of mass  $0.1 \text{ kg}$  moving on a smooth horizontal table with constant speed  $v \text{ ms}^{-1}$  describes a circle with centre  $O$  and radius  $r \text{ m}$ . The particle is attracted towards  $O$  by a force of magnitude  $4v \text{ N}$  and repelled from  $O$  by a force of magnitude  $\frac{k}{r} \text{ N}$ , where  $k$  is a constant. Given that  $k = 30$  and  $r = 1$ , find the possible values of  $v$ .

$10\text{ms}^{-1}$  or  $30\text{ms}^{-1}$

7. Two light inextensible strings  $AB$  and  $BC$  each of length  $l$  are attached to a particle of mass  $m$  at  $B$ . The other ends  $A$  and  $C$  are fixed to two points in a vertical line such that  $A$  is a distance  $l$  above  $C$ . The particle describes a horizontal circle with constant angular velocity  $\omega$ . Find

(a) the tension in the strings

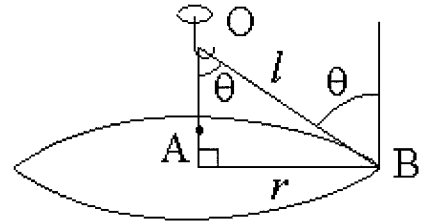


$$m\left(\frac{l\omega^2}{2} - g\right), m\left(\frac{l\omega^2}{2} + g\right)$$

(b) the least value of  $\omega$  in order that strings are taut.

$$\left(\frac{2g}{l}\right)^{1/2}$$

8. Two particles  $A$  and  $B$  of masses  $m$  and  $\mu$  respectively are attached to the ends of a light inextensible string which passes over a smooth hook at  $O$  which is free to rotate. The particle  $A$  hangs at rest vertically below  $O$  while the particle  $B$  moves in a horizontal circle with constant speed  $v$ . Find expressions for  
 (a) the length  $OB$  and the angle  $AOB$



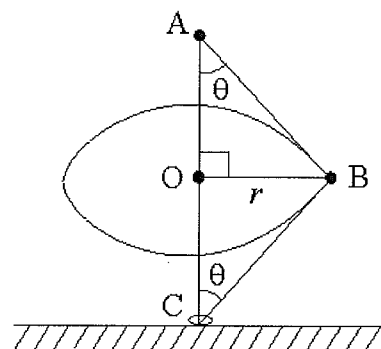
- (b) the radius of the circle in which  $B$  moves.

$$\frac{m\mu v^2}{g(m^2 - \mu^2)}, \cos^{-1}\left(\frac{\mu}{m}\right)$$

$$\frac{\mu v^2}{g\sqrt{m^2 - \mu^2}}$$



9. Two rigid light rods  $AB$  and  $BC$ , each of length  $0.5\text{ m}$ , are smoothly jointed at  $B$  and the rod  $AB$  is smoothly jointed at  $A$  to a fixed smooth vertical rod. The joint at  $B$  has a particle of mass  $2\text{ kg}$  attached. A small ring of mass  $1\text{ kg}$  is smoothly jointed to  $BC$  at  $C$  and can slide on the vertical rod below  $A$ . The ring rests on a smooth horizontal ledge at a distance  $\frac{\sqrt{3}}{2}\text{ m}$  below  $A$ . The system rotates about the vertical rod with constant angular velocity  $6\text{ radians per second}$ . Find  
 (a) the forces in the rod  $AB$  and  $BC$



$$18 + \frac{20}{\sqrt{3}}\text{ N}, 18 - \frac{20}{\sqrt{3}}\text{ N}$$

(b) the force exerted by the ledge on the ring.

$$20 - 9\sqrt{3}\text{ N}$$