Topic 23: Exercises on Motion in a Horizontal Circle
Level 3

1. A particle of mass 0.25 *kg* is attached to one end of light inextensible string of length 0.5 *m*. The other end is fixed to a point *A* on a smooth horizontal table. The particle is set in motion in a circular path. If the string breaks when the tension in it exceeds 50 N, find the greatest angular velocity at which the particle can travel.

$20 rad\, s^{-1}$

2. A mass of 1 *kg* is fastened by a string of length 1 *m* to a point 0.5 *m* above a smooth horizontal table and is describing a circle on the table with uniform angular speed of 1 revolution in 2 seconds. Find the force exerted on the table and the tension in the string.

$g - \frac{\pi^2}{2}$N, π^2N

3. One end of a light inextensible string of length l is attached to a fixed point O which is at a height $\frac{1}{3}l$ above a smooth horizontal table. A particle of mass m is attached to the other end of the string and rests on the table with the string taut. The particle is set in motion so that it moves in a circle on the table with constant speed v.

(a) Find the tension in the string and the reaction exerted on the particle by the table

$$\boxed{\frac{9mv^2}{8l},\ m\left(g-\frac{3v^2}{8l}\right)}$$

(b) Show that $v^2 \le \frac{8gl}{3}$.

4. The base of a hollow cone of semi-vertical angle 30^0 is fixed to a horizontal table. Two particle each of mass m are connected by a light inextensible string which passes through a small smooth hole in the vertex C of the cone. One particle A hangs at rest inside the cone while the other particle B moves on the outer smooth surface of the cone in a horizontal circle with centre A. Find

(a) the tension in the string and the normal reaction of the cone on B

$T = mg$, $N = mg\left(2-\sqrt{3}\right)$

(b) the angular velocity of B.

$\left(\frac{2g}{l}\left(2-\sqrt{3}\right)\right)^{1/2}$

5. A light inextensible string of length $5l$ has one end fixed at a point A and the other end fixed at a point B which is at a distance $4l$ vertically below A. A particle P of mass m is fastened to the midpoint of the string and moves with speed v, with the parts AP and BP of the string both taut, in a horizontal circular path whose centre is the midpoint of AB.

(a) Find the tension in the two parts of the string.

$AP : \frac{5}{8}mg + \frac{5}{9}\frac{mv^2}{l}, \quad BP : \frac{5}{9}\frac{mv^2}{l} - \frac{5}{8}mg$

(b) Show that the motion described can take place only if $8v^2 > 9gl$.

6. A particle A of mass $2m$ is attached by a light inextensible string of length l to a fixed point O and is also attached by another light inextensible string of the same length to a small ring B of mass $3m$ which can slide on a fixed smooth vertical wire passing through O. The particle A describes a horizontal circle with OA inclined at an angle $\frac{\pi}{3}$ with the downward vertical. Find

(a) the tension in the strings

$OA : 10mg,\ AB : 6mg$

(b) the angular velocity of A.

$\sqrt{\frac{8g}{l}}$

7. Two light rigid rods AB and BC, each of length $2m$, are smoothly jointed at B and the rod AB is smoothly jointed at A to a fixed smooth vertical rod. The joint at B has a particle of mass 2 kg attached. A small ring of mass 1 kg is smoothly jointed to BC at C and can slide on the vertical rod below A. The ring rests on a smooth horizontal ledge fixed to the vertical rod at a distance $2\sqrt{3}\ m$ below A. The system rotates about the vertical rod with constant angular velocity ω.

(a) Find the forces in the rods and the force exerted on the ring by the ledge

$AB: 2\omega^2 + \frac{2g}{\sqrt{3}}$, $BC:\ 2\omega^2 - \frac{2g}{\sqrt{3}}$, $2g - \sqrt{3}\omega^2$

(b) What happens to the system when $\omega^2 > \frac{2g}{\sqrt{3}}$?

the ring lifts off the ledge

8. A small ring C cam move freely on a light inextensible string. The two ends of the string are attached to points A and B, where A is vertically above B and at a distance c from it. When the ring C is describing a horizontal circle with constant angular velocity ω, the distances of C from A and B are b and a respectively. Show that $2gc(a+b)=\omega^2(a-b)\left\{c^2-(a+b)^2\right\}$.

9. A particle hangs by a light inextensible string of length l from a fixed point O. A second particle of the same mass hangs from the first particle by a second string of the same length. The whole system moves with constant angular velocity ω about the vertical through O, the upper and lower strings making angles α and β respectively with the vertical. Show that

(a) $\tan\alpha = \dfrac{l\omega^2}{g}\left(\sin\alpha + \dfrac{1}{2}\sin\beta\right)$

(b) $\tan\beta = \frac{l\omega^2}{g}(\sin\alpha + \sin\beta)$