Topic 10: Exercises on the Rectangular Hyperbola
Level 1

1. For the rectangular hyperbola $xy = 8$, find (a) the eccentricity, (b) the coordinates of the foci, (c) the equations of the directrices, (d) the equations of the asymptotes. Sketch the hyperbola.

(a) $\sqrt{2}$; (b) $(4,4)$, $(-4,-4)$; (c) $x + y = \pm 4$; (d) $x = 0$, $y = 0$

2. For the rectangular hyperbola $xy = 4$, find the parametric equation.

$x = 2t,\ y = \dfrac{2}{t}$

3. For the rectangular hyperbola $x = 4t, y = \frac{4}{t}$, find the Cartesian equation.

$xy = 16$

4. Find the equations of the tangent and the normal to the rectangular hyperbola $xy = 8$ at the point $(4,2)$.

$x + 2y = 8;\ 2x - y = 6$

5. Find the equations of the tangent and the normal to the rectangular hyperbola $x = 2t, y = \frac{2}{t}$ at the point where $t = 4$.

$x + 16y = 16$; $32x - 2y = 255$

6. Find the equation of the chord of contact of tangents from the point $(2,1)$ to $xy = 10$.

$x + 2y = 20$

7. The point $P\left(ct, \frac{c}{t}\right)$, where $t \neq 1$, lies on the rectangular hyperbola $xy = c^2$. The tangent and normal at P meet the line $x = y$ at T and N respectively. Show that $OP = PN$.

8. P and Q are variable points on the rectangular hyperbola $xy = 9$. The tangents at P and Q meet at R. If PQ passes through the point $(6,2)$, find the equation of the locus of R.

$x + 3y = 9$

9. The point $P\left(ct, \frac{c}{t}\right)$ lies on the rectangular hyperbola $xy = c^2$. The tangent at P cuts the x-axis at X and the y-axis at Y. Show that the aria of ΔYOX is independent of t.