Topic 10: Exercises on the Rectangular Hyperbola
Level 2

1. For the rectangular hyperbola $xy = 16$, find (a) the eccentricity; (b) the coordinates of the foci; (c) the equations of the directrices; (d) the equations of the asymptotes. Sketch the hyperbola.

(a) $\sqrt{2}$; (b) $(4\sqrt{2}, 4\sqrt{2})$, $(-4\sqrt{2}, -4\sqrt{2})$; (c) $x + y = \pm 4\sqrt{2}$; (d) $x = 0$, $y = 0$

2. Find the parametric equation of the rectangular hyperbola $xy = 25$.

$x = 5t,\ y = \dfrac{5}{t}$

3. Find the Cartesian equation of the rectangular hyperbola $x = 3t, y = \frac{3}{t}$.

$xy = 9$

4. Find the equations of the tangent and the normal to the rectangular hyperbola $xy = 12$ at the point $(-3,-4)$.

$4x + 3y = -24$, $3x - 4y = 7$

5. Find the equations of the tangent and the normal to the rectangular hyperbola $x = 3t, y = \frac{3}{t}$ at the point $t = -1$.

$x + y = -6$, $x - y = 0$

6. Find the equation of the chord of contact of tangents from the point $(1,-2)$ to $xy = 6$.

$2x - y = -12$

7. Find the equation of the chord of contact of tangents from the point $(-1,-3)$ to the rectangular hyperbola $xy = 4$. Hence find the coordinates of their points of contact and the equations of these tangents.

$3x + y = -8$, $(-2,-2)$, $\left(-\frac{2}{3},-6\right)$

8. The points $P\left(cp, \frac{c}{p}\right)$ and $Q\left(cq, \frac{c}{q}\right)$ lie on the rectangular hyperbola $xy = c^2$. The chord PQ subtends a right angle at the another point $R\left(cr, \frac{c}{r}\right)$ on the hyperbola. Show that the normal at R is parallel to PQ.

9. The point $P\left(ct, \frac{c}{t}\right)$, where $t \neq 1$ lies on the rectangular hyperbola $xy = c^2$. The tangent and the normal at P meet the line $y = x$ at T and N respectively. Show that $OT \cdot ON = 4c^2$.

10. On the rectangular hyperbola $xy = c^2$ there are variable points P and Q. The tangents at P and Q meet at R. Find the equation of the locus of R if PQ passes through the point $(a, 0)$.

$$y = \frac{2c^2}{a}$$

11. The point $P\left(ct, \frac{c}{t}\right)$ lies on the rectangular hyperbola $xy = c^2$. The tangent at *P* cuts the *x*-axis at *X* and the *y*-axis at *Y*. Show that $PX = PY$.