Part 1

## PAST EXAMINATION QUESTIONS Tangents and Normals

1. Find the co-ordinates of the point of the curve $y = x^3 + x^2 - 3x + 1$ at which the tangent is parallel to the tangent at $\left(\frac{1}{3}, \frac{4}{27}\right)$. (N61/P1/3i)

2. Prove that the curves $y^2 = 16x$ and $y = x^2 + 3$ touch at the point (1, 4). Prove also that the common tangent at this point forms with the axes a triangle of unit area. (J67/P2/2)

3. Given that the curve $y = ax^2 + bx + c$ cuts the $x$-axis at the points, $P$, (1, 0) and $Q$, (2, 0), and cuts the $y$-axis at the point $R$, (0, –4), find the numerical values of $a$, $b$ and $c$. Prove that the tangent to the curve at $P$ is parallel to the line $QR$. (N69/P2/1)

4. The straight line $x = 3$ intersects the circle $x^2 + y^2 - 8x - 10y - 9 = 0$ at the points $P$ and $Q$. Calculate (a) the co-ordinates of $P$ and $Q$, (b) the equations of the tangents at $P$ and $Q$, (c) the co-ordinates of the point of intersection of the tangents. (J70/P2/8)

5. The line $y = \frac{1}{2}$ meets the portion of the curve $y = \sin 2x$ for which $0 < x < \frac{1}{2}\pi$ in points $A$ and $B$. Calculate the gradients of the tangents to the curve at $A$ and $B$. Show that if these tangents meet at $C$ then $ABC$ is an equilateral triangle. (N74/P1/15)

6. $P$ is the point (4, 7) on the curve $y = x^2 - 6x + 15$. Find the gradient of the curve at $P$, and the equation of the tangent at this point. The tangent at another point $Q$ is perpendicular to the tangent at $P$. Calculate the $x$ co-ordinate of $Q$. (N74/P2/8)

7. Find the equation of the tangent to the curve $y = x^2 + c$ at the point where $x = k$. Deduce the relation between $k$ and $c$ if this tangent passes through the origin. For the case when $c = 4$ find the equations of the tangents to the curve which pass through the origin. (N75/P2/16a)

8. Find the equation of the normal to the curve $y = \frac{x-2}{2x+1}$ at the point where the curve crosses the $x$-axis. (J76/P2/15)

9. Find the gradient of the normal to the curve $y = x^2\sqrt{1+4x}$ at the point (2, 12). (N76/P1/16b)

10. A curve is defined by $y = x^3 - 12x$. Find the equation of the tangent to the curve at the point (3, –9) (N76/P2/1ii)

1. $(-1, 4)$
3. $a = -2, b = 6, c = -4$
4. (a) $(3, 12)(3, -2)$
   (b) $x - 7y + 81 = 0,$
   $x + 7y + 11 = 0$
   (c) $(-46, 5)$
5. $\pm\sqrt{3}$
6. $2; y = 2x - 1; 2\frac{3}{4}$
7. $y = 2kx - k^2 + c, c = k^2, y = \pm 4x$
8. $y = -5x + 10$
9. $-\frac{3}{44}$
10. $y = 15x - 54$