Past Examination Questions

Tangents and Normals

Part 2

1. Show that the tangent to the curve $y=(x+k)^2$ at the point where $x=2k$ is $y+3k^2=6kx$. This tangent meets the x-axis at P and the y-axis at Q. The mid-point of PQ is M. Find the co-ordinates of M in terms of k and hence deduce the equation of the locus of M as k varies. (J77/P2/14)

2. Find the equation of the tangent to the curve $y=\frac{4}{x^3}$ at the point where $x = 2$. This tangent meets the axes at P and Q, and R is the mid-point of PQ. Find the co-ordinates of R and determine whether or not R lies on the curve $y=\frac{4}{x^3}$. (N77/P1/6)

3. Given the curve $y=\frac{18}{x}$, find the equation of the normal at the point (6, 3). (N77/P2/14i)

4. Find the equation of the tangent to the curve $y=x^3-7x^2+14x-8$ at the point where $x = 1$. Find the x co-ordinate of the point at which the tangent is parallel to the tangent at $x = 1$. (J78/P2/6)

5. Given the curve $y=\frac{5}{3}x+kx^2-\frac{8}{9}x^3$, calculate the possible values of k such that the tangents at the points with x co-ordinates 1 and $-\frac{1}{2}$ respectively are perpendicular. (N79/P2/2)

6. Find the equation of the tangent to the curve $y=3x^2$ at the point where $x = h$. If this tangent meets the y-axis at P find, in terms of h, the y co-ordinate of P. (J80/P2/16a)

7. Find (i) the equation of the tangent to the curve $y=x-\frac{2}{x}$ at the point (2, 1), (ii) the area of the triangle enclosed by this tangent and the co-ordinate axes. (N80/P2/2)

8. Show that the equation of the tangent to the curve $y=(x+2a)^3$ at the point where $y=a^3$ is $y=3a^2x+4a^3$. This tangent meets the x-axis at P and the y-axis at Q. Find the co-ordinates of M, the midpoint of PQ, in terms of a. Deduce that, whatever the value of a, M lies on the curve $4y+27x^3=0$ (J81/P2/16)

9. The equation of a curve is $y=2x^3-7x^2+15$. Write down an expression for $\frac{dy}{dx}$ and hence find the equation of the tangent to the curve at (2, 3). (N81/P1/3i)

10. Find, in terms of h, the equation of the tangent to the curve $y=x^2$ at the point P, whose x co-ordinate is h. This tangent intersects the x-axis at A and the y-axis at B. The midpoint of AB is Q. (i) Find the co-ordinates of Q in terms of h. (ii) Find the equation of the locus of Q as h varies. (iii) Given that $h = 4$, find the co-ordinates of the point at which PQ produced meets the locus again. (J82/P2/16)

1. $(\frac{k}{4}, -\frac{3k^2}{2})$, $y = -24x^2$
2. $4y + 3x = 8$; $(1\frac{1}{3}, 1)$; no
3. $y = 2x - 9$
4. $y = 3x - 3$, $3\frac{2}{3}$
5. 0, $\frac{3}{2}$
6. $y = 6hx - 3h^2$, $-3h^2$
7. (i) $2y = 3x - 4$
   (ii) $\frac{4}{3}$ square units
8. $(-\frac{2a}{3}, 2a^3)$
9. $6x^2 - 14x$; $y + 4x = 11$
10. $y = 2hx - h^2$
    (i) $(\frac{h}{4}, \frac{h^2}{2})$
    (ii) $y = -8x^2$
    (iii) $(-2, -32)$