Past Examination Questions

Tangents and Normals

Part 6

1. The points P and Q are on the curve $y = x^2$. The value of x at P is -2 and the value of x at Q is 3. Find the equation of (i) the tangent to the curve at P, (ii) the normal to the curve at Q. The tangent at P meets the y-axis at A and the normal at Q meets the y-axis at B. Find the distance AB. (N96/P1/3)

2. Find the gradient of the curve $y = x^2 + \frac{24}{x}$ at the point $P(2, 16)$. The tangent to the curve at P meets the x-axis at A and the y-axis at B. Calculate the area of the triangle AOB, where O is the origin. (J97/P1/4)

3. The gradient at any point on a particular curve is given by the expression $x^2 + \frac{16}{x^2}$, where $x > 0$. Given that the curve passes through the point $P(4, 18)$, find (i) the equation of the normal to the curve at P, (ii) the equation of the curve. Find the coordinates of the point on the curve when the gradient is a minimum and calculate this minimum value. (N97/P1/12)

4. A curve has the equation $y^2x^3 = 72$. Show that $\frac{dy}{dx} = \frac{-3y}{2x}$ and hence, or otherwise, find the equation of the normal to the curve at the point where $y = 3$. (J97/P2/6b)

5. A curve has the equation $y = \frac{6}{1-2x}$. Find an expression for $\frac{dy}{dx}$. Hence find (i) the equation of the normal to the curve at the point where $x = 2$, (ii) the approximate increase in y as x increases from 2 to $2 + p$, where p is small. (N98/P1/4)

6. Find the equation of the tangent to the curve $y^2 = x^2y + 6x$ at the point $(2, 6)$. (N98/P2/6b)

7. Find the value of the constant c for which the line $3y = x + c$ is a normal to the curve $y = x^2 - x + 3$. (J99/P1/10)

8. The equation of a curve is $y = 3x^2 - kx + 2$, where k is a constant. The tangent to the curve, at the point where $x = 2$, passes through $(5, 5)$. Find the value of k. (J99/P1/12b)

9. The gradient at any point (x, y) on a particular curve is given by $\frac{dy}{dx} = 1 + \frac{1}{2x^2}$. The equation of the tangent at the point P on the curve is $y = 3x + 1$. Given that the x-coordinate of P is positive, find (i) the coordinates of P, (ii) the equation of the curve. (N99/P1/7)

10. Find the equation of the tangent to the curve $y = \sqrt{x^2 - 6x + 25}$ at the point $(0, 5)$. (N99/P2/6b)

1. (i) $y + 4x + 4 = 0$

(ii) $6y + x - 57 = 0$; $AB = 13\frac{1}{2}$ units

2. -2; 100 units^2

3. (i) $17y + x = 310$

(ii) $3xy = x^4 - 48 + 2x$

(2, 8), 8

4. $9y - 4x = 19$

5. $\frac{dy}{dx} = \frac{12}{(1-2x)^2}$

(i) $4y + 3x + 2 = 0$

(ii) Approximate increase in $y \approx \frac{4}{3}p$

6. $15x - 4y = 6$

7. $c = 16$

8. $k = 9$

9. (i) $(\frac{1}{2}, 2\frac{1}{2})$

(ii) $y = x - \frac{1}{2x} + 3$

10. $5y + 3x = 25$