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SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2012
YEAR 11 Mathematics
Yearly

Mathematics

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Each Section is to be returned in a separate bundle.
- Marks may **NOT** be awarded for messy or badly arranged work.
- All necessary working should be shown in every question.
- Answer must be given in simplest exact form.
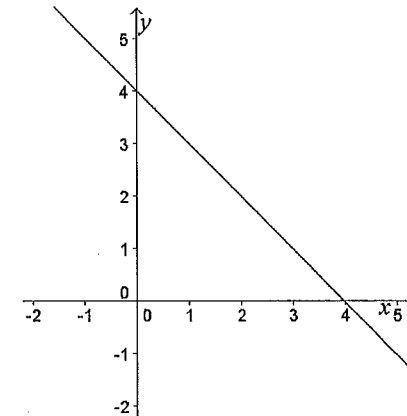
Total Marks – 70

- Attempt questions 1-15

Examiner: *P. Bigelow*

Section I (10 marks)

Answer this section on the Multiple Choice Answer Sheet

(1) The equation of the line is:

(A) $x - y + 4 = 0$

(B) $x + y - 4 = 0$

(C) $y = -x - 4$

(D) $x - y - 4 = 0$

(2) $x^2 - 4x + 6$ has a minimum value of:

(A) 4

(B) 2

(C) 6

(D) 8

(3) $3^x \times 2^x$ is equal to:

(A) 6^x

(B) 6^{2x}

(C) 5^x

(D) 5^{2x}

(4) The full solution to $x^2 = 2x$ is:

(A) $x = 2$

(B) $x = -2, 0$

(C) $x = 0, 2$

(D) $x = -2$

(5) The solution to $\log_5 x = 4$ is:

(A) 625

(B) 25

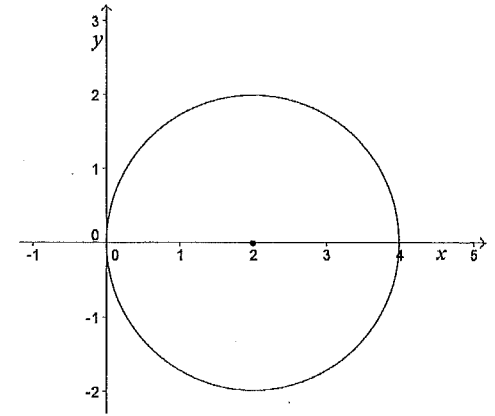(C) 125

(D) 20

(6) If $g(x) = 6 - x - x^2$ then $g(-2)$ is:

(A) 6

(B) 4

(C) 0

(D) 12

(7) If $25^{2-x} = 125$ then x is equal to:

(A) 1

(B) $\frac{1}{2}$

(C) -1

(D) 5

(8) The equation of the circle is:

(A) $x^2 + (y - 2)^2 = 4$

(B) $(x - 2)^2 + y^2 = 16$
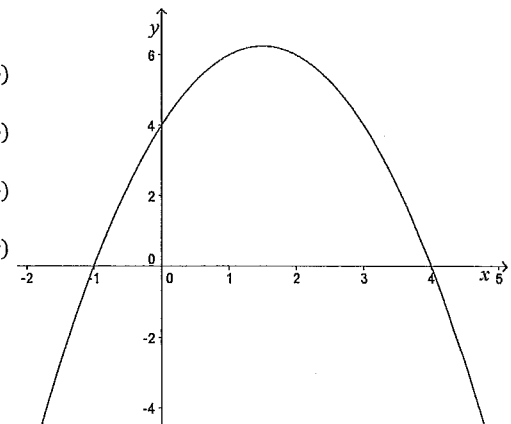
(C) $(x + 2)^2 + y^2 = 16$

(D) $(x - 2)^2 + y^2 = 4$

(9) The domain for the function $f(x) = \sqrt{2 - x}$ is:

(A) $x \geq 0$

(B) $x \geq 2$

(C) $x \leq 2$

(D) All reals

(10) The graph best represents

(A) $y = (x + 1)(x - 4)$

(B) $y = (x - 1)(x + 4)$

(C) $y = (1 - x)(x - 4)$

(D) $y = (x + 1)(4 - x)$

Section II (60 marks)

Answer this section in the Writing Booklets

Question 11   [11 marks]

(a) Sketch the following on separate number plane graphs   **[3]**

    (i) $x^2 + y^2 = 25$
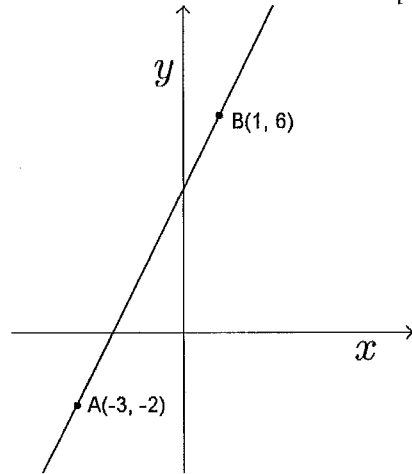
    (ii) $y = 4^x$

    (iii) $xy \leq 3$

(b) Find   **[4]**
    (i) Mid-point of AB.

    (ii) The gradient of AB.

    (iii) The equation of AB.

(c) Find the gradient of the line $2x + 3y - 11 = 0$ **[1]**

(d) The point $(-3, c)$ lies on $x + 4y - 9 = 0$, find the value of c.   **[1]**

(e) Find the perpendicular distance of the line $3x - 4y - 10 = 0$ from the origin.   **[2]**

Question 12   [12 marks]

(a) Find the value of x in the following: **[5]**

    (i) $\log_x 36 = 2$

    (ii) $x = \log_3 \frac{1}{9}$

    (iii) $\log_4 x = 3$

    (iv) $2^{2x} = 16^{x-8}$

(b) Sketch $y = \log_2 x$ for $0 < x \leq 8$.   **[2]**

(c) If $f(x) = x^2 - x$, find the values of a such that $f(a) = f(2)$.   **[2]**

(d) State whether the following functions are even (E), odd (O) or neither (N).   **[3]**

    (i) $f(x) = 2 - x^2$

    (ii) $f(x) = \dfrac{x}{1+x}$

    (iii) $f(x) = x\sqrt{x^2 + 1}$

Question 13   [12 marks]

(a) Find the solutions to the following quadratic equations (leave in simplest exact form).

[4]

    (i) $x^2 + 6x - 4 = 0$

    (ii) $6x^2 - 17x = 14$

(b) Find the centre and radius of the circle $x^2 - 6x + y^2 + 8y = 0$.   **[2]**

(c) Find   **[6]**

    (i)   Focus

    (ii) Directrix

    (iii) Vertex

for the following

    (α) $x^2 = 12y$

    (β) $x^2 - 4x + 8y = 0$

Question 14   [11 marks]

(a) Find A, B and C such that   **[3]**

$$A(x + 1)^2 + B(x + 1) + C \equiv 3x^2 + 4x + 5$$

(b) Solve the following simultaneous equations   **[3]**
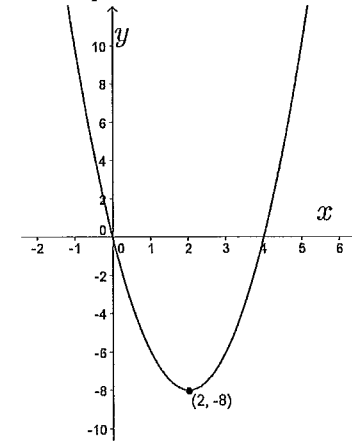
$$5^{x+y} = \frac{1}{5}$$

$$5^{3x+2y} = 1$$

(c) Find the equation of the parabola.   **[2]**

(d) Find the equation of the line perpendicular to $2x + y - 5 = 0$ and passing through $(5, 1)$.   **[3]**

Question 15   [14 marks]

(a) If the roots of $x^2 - 2x - 4 = 0$ are α and β find the value of: [5]

    (i) $\alpha + \beta$

    (ii) $\alpha\beta$

    (iii) $\frac{1}{\alpha} + \frac{1}{\beta}$

    (iv) $\alpha^2 + \beta^2$

(b) For what values of k does $x^2 - (k + 5)x + 9 = 0$ have   [3]

    (i)   Equal roots

    (ii) No real roots

(c) Form a quadratic equation with roots $2 + \sqrt{3}$ and $2 - \sqrt{3}$.   [2]

(d) The cost of producing y items per day is $\$\left(\frac{1}{3}y^2 + 45y + 27\right)$. The price at which each item may be sold is $\$\left(60 - \frac{1}{2}y\right)$.   [4]

    (i)   Find an expression for the daily profit.

    (ii) Find the maximum value of the profit.

End of Exam

Mathematics

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 =

(A) 2   (B) 6   (C) 8   (D) 9

A ◯   B ●   C ◯   D ◯

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ●   B ✖   C ◯   D ◯

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.
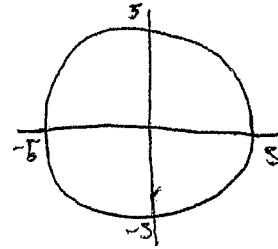
↗ *correct*

A ✖   B ✖   C ◯   D ◯

Section I: Multiple choice answer sheet.

Completely colour the cell representing your answer. Use black pen.

1. Ⓐ ⬤Ⓑ Ⓒ Ⓓ
2. Ⓐ ⬤Ⓑ Ⓒ Ⓓ
3. ⬤Ⓐ Ⓑ Ⓒ Ⓓ
4. Ⓐ Ⓑ ⬤Ⓒ Ⓓ
5. ⬤Ⓐ Ⓑ Ⓒ Ⓓ
6. Ⓐ ⬤Ⓑ Ⓒ Ⓓ
7. Ⓐ ⬤Ⓑ Ⓒ Ⓓ
8. Ⓐ ⬤Ⓑ Ⓒ Ⓓ
9. Ⓐ Ⓑ ⬤Ⓒ Ⓓ
10. Ⓐ Ⓑ Ⓒ ⬤Ⓓ
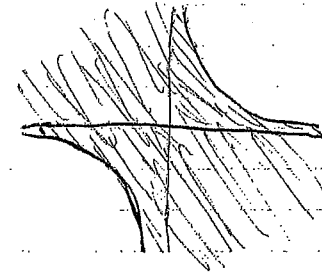
Q11 (a)
(i)

(ii)

3

(iii)

(b) (i) Mid Point $AB = \left(\dfrac{1-3}{2}, \dfrac{6-2}{2} \right)$

$= (-1, 2)$

(ii) $m_{AB} = \dfrac{6+2}{1+3}$ 4

$= 2.$

(iii) $y + 2 = 2(x+3)$

$y = 2x + 4.$

(c) $2x + 3y - 11 = 0$

$$y = -\frac{2}{3}x + \frac{11}{3}.$$

$$m = -\frac{2}{3}.$$ 1

(d) $-3 + 4c - 9 = 0$

$$4c = 12$$ 1

$$c = 3.$$

(e)

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|-10|}{\sqrt{9+16}}$$ 2

$$= 2 \text{ units}$$

Q12 (a) (i) $x = 6$

(ii) $x = -2$ 3

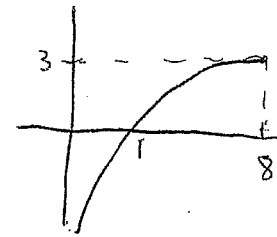(iii) $x \cdot 4^3 = 64$ $4x - 32$

(iv) $2^{2x} = 2^{4x-32}$ 2

$$2x = 4x - 32$$
$$2x = 32$$
$$x = 16$$

(b)

2

(c) $a^2 - a = 4 - 2$

$$a^2 - a - 2 = 0.$$

~~$x = \frac{1 \pm \sqrt{1 + 8}}{2}$~~   2

$$(a-2)(a+1) = 0$$
$$a = 2, -1.$$

(d) (i) Even

(ii) Neither   3

(iii) Odd.

Q13 (a) (i) $x^2 + 6x - 4 = 0.$

$$x = \frac{-6 \pm \sqrt{36 + 16}}{2} = \frac{-6 \pm \sqrt{52}}{2}$$ 2

$$= \frac{-6 \pm 2\sqrt{13}}{2}$$

$$= -3 \pm \sqrt{13}.$$

(ii) $6x^2 - 17x - 14 = 0$.

$$x = \frac{17 \pm \sqrt{17^2 - 4 \times 6 \times 14}}{12}$$

$$= \frac{17 \pm 25}{12}$$

$$= \frac{7}{2}, -\frac{2}{3}. \qquad 2$$

(b) $(x-3)^2 - 9 + (y+4)^2 - 16 = 0$

$$(x-3)^2 + (y+4)^2 = 25.$$

centre $(3, -4)$ 2
radius 5.

(c) (α) (i) $(0, 3)$
     (ii) $y = -3$.   3
     (iii) $(0, 0)$

(β) $(x-2)^2 = -8y + 4$.

$$(x-2)^2 = -4 \times 2 \left(y - \frac{1}{2}\right)$$

(i) $(2, -1\frac{1}{2})$
(ii) $y = 2\frac{1}{2}$.   3
(iii) $(2, \frac{1}{2})$

Q14
(a) $A(x+1)^2 + B(x+1) + C \equiv 3x^2 + 4x + 5$.

when $x = -1$.

$$C = 3 - 4 + 5$$
$$= 4$$

$\underline{x^2}$ $A = 3$.

$\underline{\text{constant}}$.
     $A + B + C = 5$
     $3 + B + 4 = 5$.
       $B = -2$.

$A = 3, B = -2, C = 4$.   3

(b) $x + y = -1$ ⊛

$3x + 2y = 0$.   Ⓐ
$3x + 3y = -3$.   Ⓑ
Ⓑ $-$ Ⓐ
    $y = -3$.   3
sub into $x = 2$.
   1st

(c) $y = a(x-4)x$. $(2, -8)$

$-8 = a(-2)2$.

$a = 2$.

$y = 2x(x-4)$

$y = 2x^2 - 8x$.   2

(d) $y = -2x + 5$

$m_1 = -2$

$m_1 \times m_2 = -1$

$m_2 = \frac{1}{2}$. $(5, 1)$

$y - 1 = \frac{1}{2}(x-5)$.

$2y - 2 = x - 5$ 3

$x - 2y - 3 = 0$

Q15 (a)(i) $\alpha + \beta = 2$

(ii) $\alpha\beta = -4$

(iii) $\frac{\alpha + \beta}{\alpha\beta} = \frac{2}{-4} = -\frac{1}{2}$ 3

(iv) $(\alpha+\beta)^2 - 2\alpha\beta = 4 + 8$

$= 12$ 2

(b) $x^2 - (k+5)x + 9 = 0$

(i) $\Delta = 0$

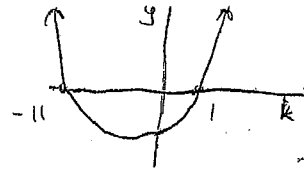$\Delta = (k+5)^2 - 4 \times 9$.

$= k^2 + 10k + 25 - 36$

$= k^2 + 10k - 11$

$k^2 + 10k - 11 = 0$.

$(k+11)(k-1) = 0$.

$k = -11, 1$.   1

(ii) $\Delta < 0$.

$-11 < k < 1$.   2

(c) $(x - 2 - \sqrt{3})(x - 2 + \sqrt{3}) = 0$.

$(x - (2+\sqrt{3}))(x - (2-\sqrt{3})) = 0$.

$x^2 - 4x + (2+\sqrt{3})(2-\sqrt{3}) = 0$ 2

$x^2 - 4x + 4 - 3 = 0$

$x^2 - 4x + 1 = 0$.

(d) (i) $P = (60 - \frac{1}{2}y) - (\frac{1}{3}y^2 + 45y + 27)$

Profit $= 33 - \frac{91}{2}y - \frac{1}{3}y^2$.

(ii) $y = \dfrac{\frac{91}{2}}{-\frac{2}{3}}$.

$= -\frac{273}{4}$.

Max Profit $= \$1\,585.69$.

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