Topic 19A: Exercises on Harder 3 Unit Induction
Level 1, Part 2

1. Show that for $n \geq 4$, $n! \geq 2^n$.

2. Show that for $n \geq 2$, $\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}<1-\frac{1}{n}$.

3. If $u_n = 5^n + 12n - 1$, show that $u_{n+1} = 5u_n - 48n + 16$ and hence show that u_n is divisible by 16 for $n \geq 1$.

4. If $u_n = 9^{n+1} - 8n - 9$, show that $u_{n+1} = 9u_n + 64n + 64$, and hence show that u_n is divisible by 64 for $n \geq 1$.

5. If $u_n = 2^{n+2} + 3^{2n+1}$, show that $u_{n+1} = 2u_n + 7 \cdot 3^{2n+1}$, and hence show that u_n is divisible by 7 for $n \geq 1$.

6. If $u_1 = 5$, $u_2 = 11$ and $u_n = 4u_{n-1} - 3u_{n-2}$ for $n \geq 3$, show that $u_n = 2 + 3^n$ for $n \geq 1$.

7. If $u_1 = 8$, $u_2 = 20$ and $u_n = 4u_{n-1} - 4u_{n-2}$ for $n \geq 3$, show that $u_n = (n+3)2^n$ for $n \geq 1$.

8. If $u_1 = 1$ and $u_n = \sqrt{2u_{n-1}}$ for $n \geq 2$

(a) show that $u_n < 2$ for $n \geq 1$,

(b) deduce that $u_{n+1} > u_n$ for $n \geq 1$.