Topic 19A: Exercises on Harder 3 Unit Induction
Level 2, Part 1

1. Show that for $n \geq 1$, $1^2 + 2^2 + \ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$.

2. Show that if $x \neq 1$, then for $n \geq 1$, $1 + x + x^2 + ... + x^n = \dfrac{x^{n+1} - 1}{x - 1}$.

3. Show that for $n \geq 1$ $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \cdots + n \cdot n! = (n+1)! - 1$.

4. Show that for $n \geq 1$ $1 \cdot \ln\frac{2}{1} + 2 \cdot \ln\frac{3}{2} + \ldots + n \cdot \ln\left(\frac{n+1}{n}\right) = \ln\left(\frac{(n+1)^n}{n!}\right)$.

5. Show that for $n \geq 1$ $\dfrac{d^n}{dx^n}\ln(1+x) = \dfrac{(-1)^{n-1}(n-1)!}{(1+x)^n}$.

6. Show that $n \geq 2$ lines, no two of which are parallel and no three of which are concurrent, have $\frac{n(n-1)}{2}$ points of intersection.

7. Show that $(1+x)^n - 1$ is divisible by x for $n \geq 1$.

8. Show that for $n \geq 5, 2^n > n^2 + 2$.