

**Topic 19A: Exercises on Harder 3 Unit Induction**  
**Level 2, Part 2**

1. Show that for  $n \neq 3$ ,  $3^n > n^n$ .

2. Show that for  $n \geq 3$   $\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} > \frac{3}{5}$ .

3. Show that for  $n \geq 1$   $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}} < 2\sqrt{n}$ .

4. If  $u_n = 3^{4n+2} + 2 \cdot 4^{3n+1}$ , show that  $u_n$  is divisible by 17 for  $n \geq 1$ .

5. Show that  $7^n + 11^n$  is divisible by 9 for odd  $n \geq 1$ .

6. If  $u_1 = 2$ ,  $u_2 = 16$  and  $u_n = 8u_{n-1} - 15u_{n-2}$  for  $n \geq 3$ , show that  $u_n = 5^n - 3^n$  for  $n \geq 1$ .

7. If  $u_n = 5^n - 4n - 1$ , show that  $u_n > 0$  for  $n \geq 2$ .

8. If  $u_1 = 1$  and  $u_n = \sqrt{3 + 2u_{n-1}}$  for  $n \geq 2$

(a) show that  $u_n < 3$  for  $n \geq 1$ ,

(b) deduce that  $u_{n+1} > u_n$  for  $n \geq 1$ .