Topic 19A: Exercises on Harder 3 Unit Induction
Level 3, Part 1

1. Show that for $n \geq 1$, $1^3 + 2^3 + 3^3 + ... + n^3 = \frac{n^2(n+1)^2}{4}$.

2. Show that for $n \geq 1$, $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ldots+\frac{n}{(n+1)!}=1-\frac{1}{(n+1)!}$.

3. Show that for $n \geq 1$, $1 + \frac{x}{1!} + \frac{x(x+1)}{2!} + \ldots + \frac{x(x+1)\ldots(x+n-1)}{n!} = \frac{(x+1)(x+2)\ldots(x+n)}{n!}$.

4. Show that for $n \geq 1$, $2 \cdot 1! + 5 \cdot 2! + 10 \cdot 3! + \ldots + (n^2 + 1)n! = n(n+1)!$.

5. Show that $(1+x)^n - nx - 1$ is divisible by x^2 for $n \geq 2$.

6. Show that for $n \geq 10, 2^n > n^3$.