Topic 19A: Exercises on Harder 3 Unit Induction
Level 3, Part 2

1. Show that for $n \geq 2, n^n > (n+1)^{n-1}$.

2. Show that for $n \geq 1, 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{n}} > 2\left(\sqrt{n+1} - 1\right)$.

3. Show that for $n \geq 1, \frac{1}{(1!)^2} + \frac{1}{(2!)^2} + \ldots + \frac{1}{(n!)^2} \leq \frac{4}{3}\left(1 - \frac{1}{4^n}\right)$.

4. Show that $35^n + 3 \cdot 7^n + 2 \cdot 5^n + 6$ is divisible by 12 for $n \geq 0$.

5. Show that $7^n + 15^n$ is divisible by 11 for odd $n \geq 1$.

6. If $u_1 = 1, u_2 = 1$ and $u_n = u_{n-1} + u_{n-2}$ for $n \geq 3$, show that for $n \geq 1$

$$u_n = \frac{1}{\sqrt{5}} \left\{ \left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right\}.$$