Topic 19: Exercises on Harder 3 Unit Inequalities
Level 1, Part 1

1. If $0 < a < b$ show that $a < \dfrac{a+b}{2} < b$.

2. If $0 < a < b$ show that $a < \sqrt{ab} < b$.

3. If $a > 0$, $b > 0$ show that $\frac{a+b}{2} > \sqrt{ab}$.

4. If $x > 0$ show that $x + \frac{1}{x} \geq 2$.

5. If $a > 0$, $b > 0$ show that $4ab \leq (a+b)^2$.

6. If $a > 0$, $b > 0$ and $c > 0$ show that $(a+b)(b+c)(c+a) \geq 8abc$.

7. If $a > 0$, $b > 0$, $c > 0$ and $a + b + c = 1$ show that $(1-a)(1-b)(1-c) \geq 8abc$.

8. If $a>0$, $b>0$ and $a+b=1$, show that $\frac{1}{a}+\frac{1}{b}\geq 4$. Hint: Show that $a+b\geq 2\sqrt{(ab)}$.

9. If $a > 0$, $b > 0$ and $a + b = 1$, show that $\frac{1}{a^2} + \frac{1}{b^2} \geq 8$. Hint: Show that $a + b \geq 2\sqrt{(ab)}$.

10. Show that $ab + bc + ca \leq a^2 + b^2 + c^2$.

11. If $a > 0$ and $b > 0$, show that $a^3 + b^3 \geq a^2 b + ab^2$.