Topic 19: Exercises on Harder 3 Unit Inequalities
Level 1, Part 2

1. Show that $(a+b)^2 \leq 2(a^2+b^2)$.

2. Show that $(a^3+b^3)^2 \leq (a^2+b^2)(a^4+b^4)$.

3. Show that $(a+b+c)^2 \le 3(a^2+b^2+c^2)$.

4. If $a > 0$, $b > 0$, $c > 0$ and $в > 0$, show that $\frac{a+b+c+d}{4} \geq \sqrt[4]{(abcd)}$. Hint: if $a > 0$ and $b > 0$, show that $a + b \geq 2\sqrt{ab}$.

5. Show that $e^x > 1 + x$ $(x > 0)$.

6. If $0 < t < 1$ show that $\frac{1}{2} < \frac{1}{1+t} < 1$. By integrating between 0 and u, deduce that for $0 < u < 1$, $\frac{u}{2} < \ln(1+u) < u$.

7. Show that $x-\frac{1}{2}x^2<\ln(1+x)<x-\frac{1}{2}x^2+\frac{1}{3}x^3$ $(x>0)$.

8. If $0 < t < 1$, show that $\frac{1}{2} < \frac{1}{1+t^2} < 1$. By integrating between 0 and u, deduce that $\frac{u}{2} < \tan^{-1} u < u$ for $0 < u < 1$.

9. If $b \geq a > 0$, show that $\frac{1}{b}(b-a) \leq \ln \frac{b}{a} \leq \frac{1}{a}(b-a)$.