Topic 19: Exercises on Harder 3 Unit Inequalities
Level 2, Part 2

1. If $a > 0$, $b > 0$ and $c > 0$, show that $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq 3$.

2. If $a > 0$, $b > 0$, $c > 0$ and $d > 0$ show that $(b+c+d)(a+c+d)(a+b+d)(a+b+c) \geq 81abcd$.

3. If $a > 0$, $b > 0$, $c > 0$ and $d > 0$ show that $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} + \frac{d}{a} \geq 4$.

Hint: Show that $\frac{a+b+c+d}{4} \geq \sqrt[4]{abcd}$.

4. Show that for $x > 0$ $\quad x > \dfrac{3\sin x}{2+\cos x}$.

5. If $t > 0$, show that $\frac{1}{(1+t)^2} < \frac{1}{1+t} < 1$. By integrating between 0 and u deduce that

$\frac{u}{1+u} < \ln(1+u) < u$.

6. Show that $\frac{1}{3}\int_0^1 x^2(1-x)^2\,dx < \int_0^1 \frac{x^2(1-x)^2}{x+2}\,dx < \frac{1}{2}\int_0^1 x^2(1-x)^2\,dx$, and hence deduce that $\frac{2627}{6480} < \ln\frac{3}{2} < \frac{2628}{6480}$.

7. Show that for $x > 0$ $e^x < 1 + x + \frac{x^2}{2} e^x$.

8. If $0 < a < 1$ and $x > -1$, show that $(1+x)^a \le 1 + ax$.

9. If $b > a > 0$, show that $\sqrt{ab} < \dfrac{b-a}{\ln b - \ln a}$.