Topic 19: Exercises on Harder 3 Unit Projectile Motion
Level 3

1. A particle is projected from a point O with speed V and angle of elevation α. At a certain point P on its trajectory, the direction of motion of the particle and the line OP are inclined (in opposite senses) at equal angles β to the horizontal. Show that

(a) the time taken to reach P from O is $\dfrac{4V\sin\alpha}{3g}$.

(b) $3\tan\beta = \tan\alpha$.

2. A particle is projected from a point O with speed $40\ ms^{-1}$ at an angle of elevation α, where $\tan\alpha = \frac{3}{4}$. Two seconds later, a second particle is projected from O and it collides with the first particle one second after leaving O. Find the initial velocity of the second particle.

$32\cdot\sqrt{10}\ ms^{-1}$ at an angle of elevation $\tan^{-1}\frac{1}{3}$

3. O is a point on horizontal ground. D is a point a distance d vertically above O. A particle is projected from O with speed U at the angle of elevation α. Simultaneously a second particle is projected horizontally from D with speed V on the same side of OD as A and in the same vertical plane through O as the first particle. If $U = 51, V = 45, d = 60$ and $\tan\alpha = \frac{8}{15}$, show that the two particle do collide and find the time and the height above O at which this occurs. (Take $g = 10\ ms^{-2}$.)

2.5 s, 28.75 m

4. A particle is projected from a point O with speed $20\ ms^{-1}$ at an angle of elevation α. T seconds later another particle is projected from O with the same speed but at an angle of elevation β, where $\beta < \alpha$. The two particles collide at a point 24 m horizontally from O and 12 m vertically above O. Taking $g = 10\ ms^{-2}$, find the value of T.

$$\frac{6}{\sqrt{5}} - 2$$

5. A projectile is fired from a point O on level ground with speed 13 ms^{-1} at an angle of elevation α, where $\tan\alpha = \frac{12}{5}$. The projectile just clears the top of a wall in its path and then reaches a maximum height of twice the height of the wall. At the instant of projection, a target is fired horizontally from the top of the wall and continues to move horizontally with constant speed U in the plane of the path of the projectile away from O. Find the value of U, given that the projectile hits the target.

$10\left(\sqrt{2}-1\right)\ ms^{-1}$

6. A particle is projected from a point O on level ground with speed V at an angle of elevation α. The particle just clears a wall of height h at a distance d from O. Show that if the speed of projection is fixed, the particle hits the ground at a distance c beyond the wall, where $g\left\{d^2c^2+(c+d)^2h^2\right\}=2dhV^2c$.

7. A particle is projected with speed V at an angle of elevation α from a point A on the edge of a cliff of height h. Simultaneously another particle is projected with speed $2V$ at an angle of elevation β from a point B, distance d from the foot of the cliff, the trajectories of the two particles being in the same vertical plane. If the two particles collide, show that $2\sin(\beta+\gamma)=\sin(\alpha+\gamma)$, where $\gamma=\tan^{-1}\dfrac{h}{d}$.

8. A particle is projected from a point O up a plane inclined at an angle α above the horizontal. The speed of projection is V and the angle of elevation θ, where $\theta > \alpha$. Find

(a) the range of the particle on the inclined plane

$$\boxed{\frac{2V^2 \cos\theta \sin(\theta - \alpha)}{g\cos^2 \alpha}}$$

(b) the maximum range up the plane.

$$\boxed{\frac{V^2}{g(1 + \sin\alpha)}}$$

9. A particle is projected with speed V from the top of a cliff of height h above sea level. Find the greatest horizontal distance the particle can cover before landing in the sea.

$$\frac{V}{g}\sqrt{V^2 + gh}$$