

Topic 19: Exercises on Harder 3 Unit Projectile Motion & S.H.M
Level 2, Part 1

1. A particle is projected with speed V and the angle of elevation α from a point O on level ground. The horizontal range is R where $R < \frac{V^2}{g}$. Show that there are two possible angles of projection for a given R and that if α_1 and α_2 are these two angles, then

$$\alpha_1 + \alpha_2 = \frac{\pi}{2}.$$

2. Two particles are projected with speed V and the angle of elevation α_1 and α_2 from a point O on level ground. The horizontal range is R where $R < \frac{V^2}{g}$. Show that if t_1 and t_2 are the respective times of flight, and h_1 and h_2 are the greatest height reached, then

$$R = \frac{1}{2}gt_1t_2 = 4\sqrt{h_1h_2}.$$

3. A particle is projected with speed V and angle of elevation α from a point O on the edge of a cliff of height h . When the particle hits the ground at the point P its path makes an angle $\tan^{-1}(2 \tan \alpha)$ with the horizontal. Find its speed of projection and OP in terms of h and α .

$$\sqrt{\frac{2gh}{3}} \operatorname{cosec} \alpha; 2h \cot \alpha$$

4. A particle is projected from a point O at time $t = 0$ with speed V and angle of elevation α . It moves under gravity and reaches its horizontal range R at time $t = T$. If the line OP at a certain point P on its trajectory makes an angle γ below the horizontal, such that

$$\tan \gamma = \frac{1}{3} \tan \alpha, \text{ show that } t = \frac{4}{3} T.$$

5. A and B are two points on level ground a distance X apart. A particle is projected from A towards B with speed V_1 and angle of elevation θ_1 . Simultaneously another particle is projected from B towards A with speed V_2 and angle of elevation θ_2 .

(a) If the two particles are to collide, show that $V_1 \sin \theta_1 = V_2 \sin \theta_2$ and find a second condition which must also be satisfied.

(b) If $V_1 = 45$, $V_2 = 60$, $\tan \theta_1 = \frac{4}{3}$, $\tan \theta_2 = \frac{3}{4}$ and $X = 150$, show that the two particles do collide and find the time and the height above AB at which this occurs.

6. 1. Owing to the tides, the height of water in an estuary may be assumed to rise and fall with time in simple harmonic motion. At a certain place there is a danger of flooding when the height of water is above 1.25 m . One day the high tide had a height of 1.5 m at 1.00 am and the following low tide had a height of 0.5 m at 7.30 am . Assuming that the following high tide also had a height of 1.5 m , find the times that day when there was a danger of flooding.

midnight to 3.10 am , 11.50 am to 4.10 pm