Topic 21: Exercises on Resisted Motion
Level 2

1. A particle of mass m is set in motion with speed u. Subsequently the only force acting upon the particle directly opposes its motion and is of magnitude $mk\left(1+v^2\right)$ where k is a constant and v is its speed at time t.

(i) Show that the particle is brought to rest after a time $\frac{1}{k}\tan^{-1} u$

(ii) Find an expression for the distance travelled by the particle in this time.

$\frac{1}{2k}\ln\left(1+u^2\right)$

2. A particle moves in a straight line away from a fixed point O in the line such that at time t its displacement from O is x and its velocity is v. At time $t = 0$, $x = 0$ and $v = V$. Subsequently the particle is slowing down at a rate equal to kv^3, where k is a positive constant. Show that $t = \frac{x}{V} + \frac{1}{2}kx^2$.

3. A particle moves in a straight line away from a fixed point O in the line such that at time t its displacement from O is x and its velocity is v. At time $t = 0$, $x = 0$ and $v = 1$. Subsequently the particle experiences a retardation of magnitude e^{v}. Find the time t_1 for the particle to slow to half its initial and the further time t_2 for the particle to come to rest. Deduce that $\frac{t_2}{t_1} = e^{\frac{1}{2}}$.

4. A particle of mass m moves in a horizontal straight line. The only force acting on the particle is a resistance of magnitude $mk\left(v^2+c^2\right)$, where k and c are positive constants and v is the speed. If $v=2c$ when $t=0$, find the distance travelled and the time taken for the speed to be halved.

$$\frac{1}{2k}\ln\left(\frac{5}{2}\right);\ \frac{1}{kc}\tan^{-1}\frac{1}{3}$$

5. A particle of mass m is projected vertically upward under gravity with speed nV in a medium in which the resistance to motion is mk times the square of speed of the particle, where k and n are positive constants and V is the terminal velocity of the particle in the medium. Find the time of ascent of the particle and the maximum height.

$$\frac{1}{\sqrt{gk}}\tan^{-1}\sqrt{\frac{k}{g}}nV\,;\ \frac{1}{2k}\ln\left(1+\frac{k}{g}n^2V^2\right)$$

6. A particle of mass $1\ kg$ is projected vertically upward under gravity with speed $2c$ in a medium in which the resistance to motion is $\frac{g}{c^2}$ times the square of the speed, where c is a positive constant. Find the speed with which the particle returns to its starting point.

$\frac{2c}{\sqrt{5}}$

7. A particle is moving vertically downward in a medium which exerts a resistance to the motion which is proportional to the square of the speed of the particle. It is released from rest at O and its terminal velocity is V. Find the distance it has fallen below O and the time taken when its velocity is one-half of its terminal velocity.

$$\frac{V^2}{2g}\ln\frac{4}{3};\ \frac{V}{2g}\ln 3$$

8. A particle of mass m is projected vertically upward under gravity with speed u in a medium in which the resistance is mk times the speed, where k is a positive constant. If the particle reaches its greatest height H in time T, show that $u = gT + kH$.

9. A particle of mass m is projected vertically upward under gravity in a medium in which the resistance is mk times the square of the speed, where k is a positive constant. If its speed of projection is equal to the terminal velocity V in the medium, show that when it returns to the point of projection its speed is $\frac{V}{\sqrt{2}}$.