Topic 21: Exercises on Resisted Motion
Level 3, Part 1

1. A particle moves in a straight line with retardation $\frac{1}{3v^2}$ where v is its velocity at time t. Initially the particle is at a fixed point O on the line with velocity $u > 0$. Find expressions in terms of u for the time taken and the distance travelled for the particle to come to rest.

$u^3, \frac{3}{4}u^4$

2. A particle of mass m moves in a horizontal straight line away from a fixed point O in the line. The particle is resisted by a force $mkv^{\frac{3}{2}}$, where k is a positive constant and v is the speed. When $t = 0$, $v = u > 0$. Show that the particle is never brought to rest and that its distance from O is at most $\frac{2}{k}\sqrt{u}$.

3. A particle moves in a straight line away from a fixed point O in the line, such that at time t its displacement from O is x and its velocity v is given by $\frac{1}{v} = A + Bt$, for some positive constants A and B. If the retardation is $1\ ms^{-2}$ and the velocity is $80\ ms^{-1}$ when $t = 0$, find the values of A and B. Express x in terms of t, and v in terms of x.

$$x = 6400 \ln\left(1 + \frac{t}{80}\right), \quad v = 80 \cdot e^{-x/6400}$$

4. A particle of mass m moves in a horizontal straight line. The only force acting on the particle is a resistance of magnitude $mk\left(v^2 + c^2\right)$, where k and c are positive constants and v is the speed. If $v = 2c$ when $t = 0$, find the additional distance travelled and the additional time taken for the particle to come to rest since the moment when the speed was halved.

$$\frac{1}{2k}\ln 2, \quad \frac{1}{kc}\cdot\frac{\pi}{4}$$

5. A particle is moving vertically downward in a medium which exerts a resistance to the motion which is proportional to the speed of the particle. The particle is released from rest at O, and at time t its position is at a distance x below O and its speed is v. If the terminal velocity is V, show that $xg + Vv = Vgt$.